Student Code Number:_____________

Ph.D. Qualifying Exam

Dynamics and Vibrations

January, 2005

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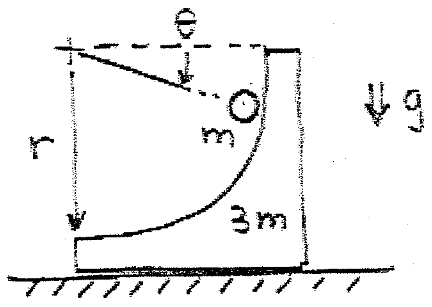
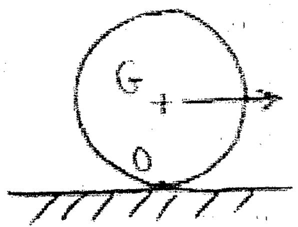
Directions: Work all four problems.
Note that the problems are EVENLY WEIGHTED.
You may use two books and two pages of notes for reference.

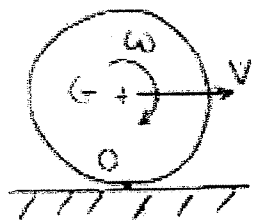
1. A particle of mass m starts at rest against the top (at $\theta = 0$) of a block of mass $3m$ which is in contact with a horizontal floor. The particle slides down the circular profile, of radius r, of the block. When the particle reaches the angle of 45° with respect to the block, the block begins to slide on the horizontal floor. The friction between the particle and the blocks is negligible. Find the static friction coefficient μ between the block and the floor.



2. A uniform sphere is initially set into pure translation at a speed v_o along a horizontal surface. The coefficient of friction between the sphere and the surface is μ. As the sphere slides, friction induces rotation, until rolling occurs. Find the loss of energy between the intial condition and the onset of rolling. The mass moment of inertia is $I_G = \frac{2}{5}mr^2$ about the mass center.



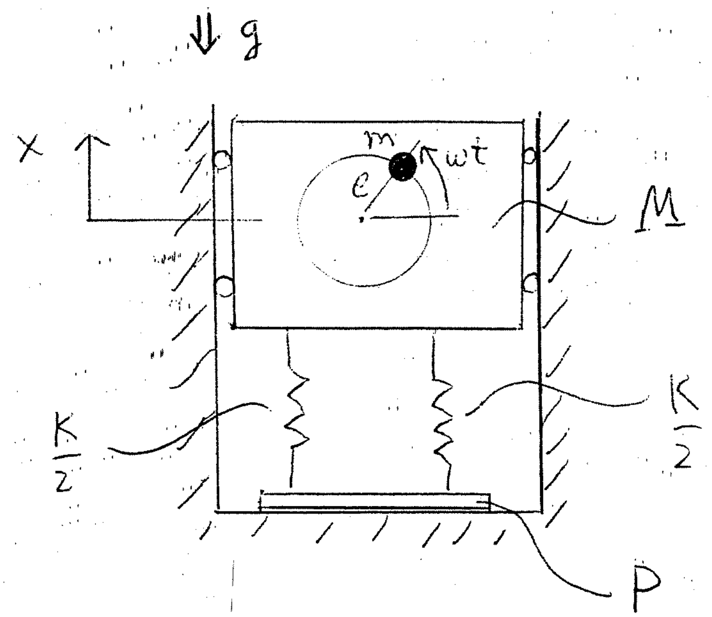
initial condition



after some sliding

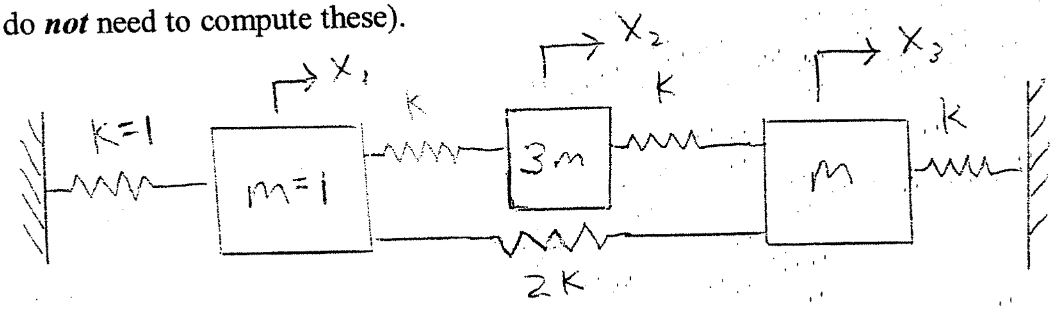
3. A single degree of freedom system of total mass M (including the eccentric mass m) is subjected to a rotating unbalance of mass m and eccentricity e, as shown below. The system is supported via springs of total stiffness k to a *massless* platform **P** that rests on the ground. The system can be assumed to be undamped. In this problem you must consider both static and dynamic effects of the forces acting on the system. The coordinate x is measured such that $x=0$ is the system's static equilibrium configuration.

Consider the steady-state system response as the rotor speed ω (that is, the excitation frequency) is very slowly increased from zero. Determine the critical speed ω_{CR} at which the platform **P** first begins to bounce off from the ground. Express your result in terms of the parameters $M, m, e, k,$ and g, where g is the gravitational acceleration.



4. Consider the three degree of freedom system with dimensionless mass m and stiffness k values as indicated in the diagram below. The natural frequencies $\omega_j\,; j = 1,2,3$ are also provided below (you do ***not*** need to compute these).

$\omega_1 = 0.528\,\text{rad/sec}$
$\omega_2 = 1.545\,\text{rad/sec}$
$\omega_3 = 2.449\,\text{rad/sec}$



(a) Determine the system mass and stiffness matrices.
(b) Determine the three system mode shapes, $\mathbf{u}_j\,; j = 1,2,3$, of the form $\mathbf{u}_j = (1 \quad \alpha \quad \beta)^{\mathrm{T}}$ where $(\bullet)^{\mathrm{T}}$ represents the transpose of $(\bullet)$. Partial answer: you should find that the mode shape associated with ω_1 is given by $\mathbf{u}_1 = (1 \quad 1.72 \quad 1)^{\mathrm{T}}$.
(c) Consider an initial configuration of the system wherein the leftmost mass is displaced by a unit amount ($x_1(0) = +1$), the inner mass is displaced by $x_2(0) = -1$, and the third is displaced by an amount $x_3(0) = a$ (to be determined). Determine the value(s) of a such that the first mode (that is, the mode associated with ω_1) is not activated by these initial conditions.
(d) Consider the steady-state system response to harmonic excitation that has equal forces applied to the end masses, that is, the force vector is given by $\mathbf{f} = (f_1 \quad f_2 \quad f_3)^{\mathrm{T}} \sin(\omega t)$ wherein $f_3 = f_1$. Prove that the resulting frequency response curves, $X_j(\omega)$ vs. ω, exhibit only **two** resonance peaks. State clearly which resonance is absent and the reason for its absence. Notation: the steady state responses of the degrees of freedom are given by $x_{j,ss} = X_j(\omega)\sin(\omega t - \phi_j)$.