Student Code Number: _____

Ph.D. Qualifying Exam

Dynamics & Vibrations

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Directions: Work all four problems.
Note that the problems are EVENLY WEIGHTED.
You may use two books and two pages of notes for reference.

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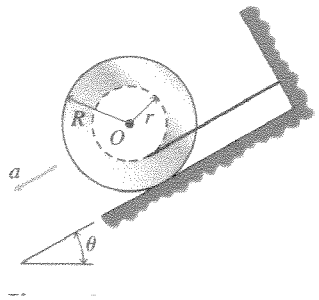
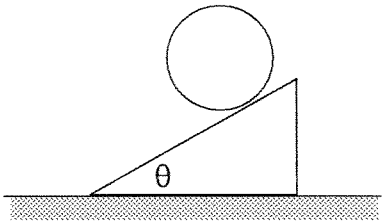
1. A cord is wrapped around a spool as shown. The coefficient of kinetic friction between the spool and the inclined plane is μ_k. The spool is released from rest. The radius of gyration of the spool is k. If the angle θ is large enough to permit motion, find the acceleration of the center O of the spool. (Consider constant downward gravity g.)



2. A uniform cylinder is rolling without sliding on a wedge. Both have mass m. The wedge is free to slide on the frictionless plane. The system is released from rest as shown. Find the velocity of the wedge after it has slid to the right by some distance h. (Consider constant downward gravity g and assume h is small enough so that the cylinder does not slide off the wedge.).



3. A two degree-of-freedom system has the following equation of motion:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} \sin(\Omega t) .$$

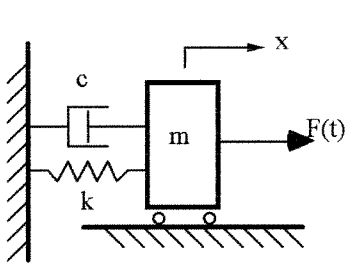
(a) Clearly state in words how you would find the natural frequencies of the system.

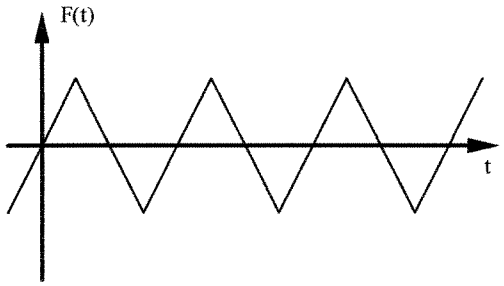
(b) For the case where m_1=1, m_2=2, k_1=1, and k_2=2, the natural frequencies are $\omega_1 = \sqrt{2-\sqrt{3}}$ and $\omega_2 = \sqrt{2+\sqrt{3}}$, find the associated mode shapes.

(c) Find the ratio of F_1/F_2 such that the mode shape associated with the lowest natural frequency can not be excited.

4. A system and its forcing function, $F(t)$, are shown below. Given that m=2 kg, k=200 N/m, and c=0.4 Ns/m (i.e., ζ=0.01), and that the forcing function has a Fourier series given by

$$F(t)=\sum_{n=1,3,5,\ldots}^{\infty}(-1)^{\frac{n-1}{2}}\frac{\sin(n2t)}{n^2}=\sin(2t)-\frac{1}{9}\sin(6t)+\frac{1}{25}\sin(10t)-\ldots\ldots$$





Answer the following.

(a) Which harmonic (i.e., "n" value) would you expect to dominate the response, $x(t)$? Clearly explain why.

(b) Estimate the amplitude of the steady-state response, clearly stating any simplifying assumptions you have made.