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Ph.D. Qualifying Exam

Dynamics and Vibrations

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S. W. Shaw and A. G. Haddow

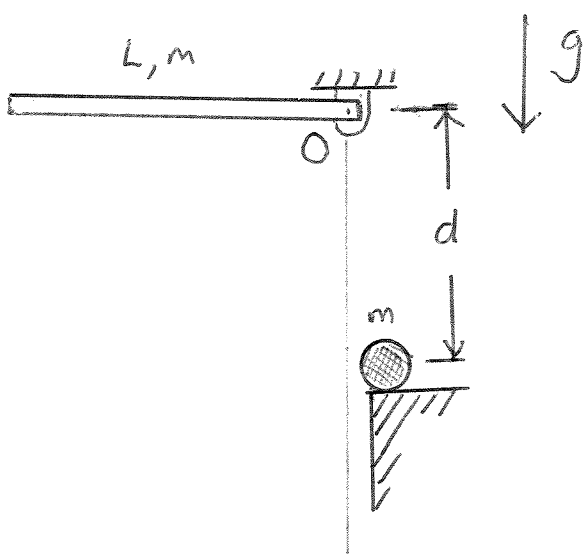
<u>Directions</u>: Work all four problems.
Note that the problems are EVENLY WEIGHTED.
You may use two books and two pages of notes for reference.

1. A uniform bar of mass m and length L rotates under the action of gravity about a frictionless pivot at point O as shown. A particle, also of mass m, rests on the corner of a smooth (frictionless) table as shown. The bar is released with zero angular velocity from the horizontal position to the left and rotates counterclockwise under the action of gravity until it strikes the particle. (The bar strikes the mass particle, not the table.) The impact occurs at a point a distance d below the pivot exactly at the instant when the bar is vertical. The collision between the bar and the particle can be modeled as a simple impact with a coefficient of restitution e. Recall that the moment of inertia of the bar about the pivot is $I = mL^2/3$.

(a) (5 points) Determine the angular velocity of the bar just before it strikes the particle; denote your result as ω_0.

(b) (15 points) Determine the angular velocity of the bar ***and*** the horizontal velocity of the particle immediately after impact; express your results in terms of the system parameters and ω_0 (you can do this even if you don't get part (a)). You should find that the particle velocity is horizontal and of magnitude $v = \dfrac{L^2 d(1+e)\omega_0}{L^2 + 3d^2}$.

(c) (5 points) Determine the value of d such that the particle velocity from (b) is maximized. Hint: let $d = \beta L$ and solve for the result in terms of the dimensionless parameter β.

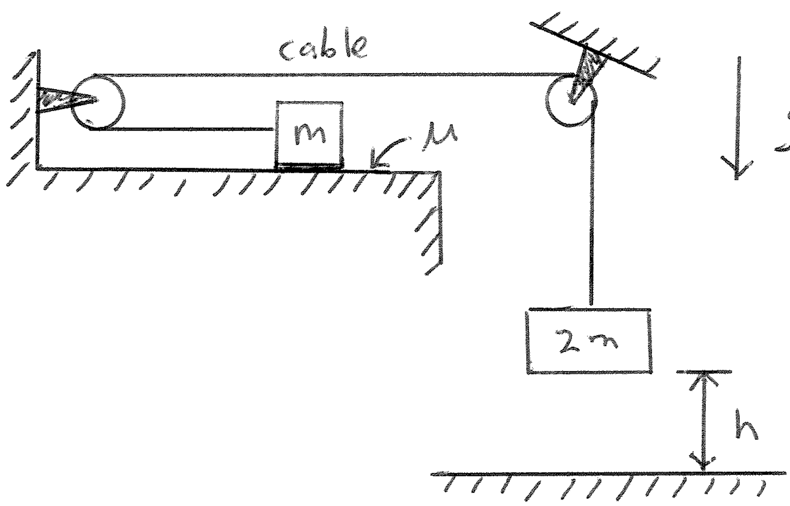


2. As shown, a block of mass m moves along a surface with sliding coefficient of friction μ. It is attached to another block of mass $2m$ via a cable that runs through two pulleys. The block of mass $2m$ starts at a height h above the floor when the system is released with zero velocity. Assume that the pulleys and cable are ideal, specifically, they are massless, frictionless, and inextensible, as appropriate. Also assume that the friction is sufficiently small that motion begins when the system is released and continues until the floor is reached.

(a) (10 points) Determine the tension in the cable as the system moves.

(b) (5 points) Determine the value of μ such that the block of mass $2m$ falls with a downward acceleration one-half that of gravity, that is, $\ddot{y} = g/2$.

(c) (10 points) Determine the speed of the $2m$ block when it hits the floor.



3. A sensitive instrument of mass 2 kg is to be attached to a very heavy structure that is vibrating with an amplitude 0.9mm at a frequency of 600rpm (see Figure 3a).

(a) (10pts) Design a spring isolation for the instrument so that no more than 10% of the heavy structure's motion (i.e., displacement) is transmitted to the instrument. You may assume very small damping.

(b) (7 pts) Independent of the answer to part (a), what is the magnitude of the steady-state acceleration of the instrument if the speed of the heavy structure were changed to 200rpm and the stiffness k=500 N/m?

(c) (8 pts) What is the magnitude of the force exerted by the instrument onto the heavy structure under the conditions given in part (b)?

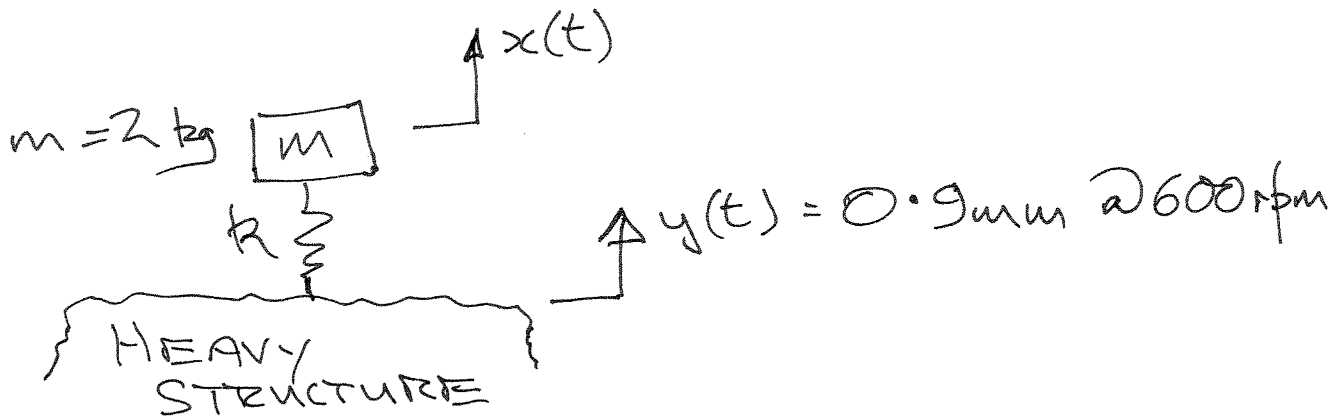


FIGURE 3a.

4. A system has the following equation of motion:

$$\begin{bmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{Bmatrix}\ddot{x}_1(t)\\ \ddot{x}_2(t)\\ \ddot{x}_3(t)\end{Bmatrix}+\begin{bmatrix}2 & -1 & 0\\ -1 & 5 & -2\\ 0 & -2 & 2\end{bmatrix}\begin{Bmatrix}x_1(t)\\ x_2(t)\\ x_3(t)\end{Bmatrix}=\begin{Bmatrix}F_1(t)\\ F_2(t)\\ F_3(t)\end{Bmatrix}.$$

The normalized modal matrix $\left[\hat{P}\right]$ is found to be: $\begin{bmatrix}0.3400 & 0.8944 & 0.2905\\ 0.4593 & 0 & -0.5376\\ 0.6801 & -0.4472 & 0.5810\end{bmatrix}$ and the natural frequencies to be $\omega_1^2=0.6492$, $\omega_2^2=2$, and $\omega_3^2=3.8508$, respectively. Using $\vec{x}(t)=\left[\hat{P}\right]\vec{p}(t)$ then yields:

$$\begin{Bmatrix}\ddot{p}_1(t)\\ \ddot{p}_2(t)\\ \ddot{p}_3(t)\end{Bmatrix}+\begin{bmatrix}\omega_1^2 & 0 & 0\\ 0 & \omega_2^2 & 0\\ 0 & 0 & \omega_3^2\end{bmatrix}\begin{Bmatrix}p_1(t)\\ p_2(t)\\ p_3(t)\end{Bmatrix}=\begin{Bmatrix}f_1(t)\\ f_2(t)\\ f_3(t)\end{Bmatrix}.$$

(a) (7pts.) Find $f_1(t)$ if $\begin{Bmatrix}F_1(t)\\ F_2(t)\\ F_3(t)\end{Bmatrix}=\begin{Bmatrix}0\\ \delta(t)\\ 0\end{Bmatrix}$, i.e., there is a unit impulse applied to the mass at co-ordinate 2.

(b) (8pts.) What is $p_1(t)$ for the forcing function given in part (a). Your answer should have no variables in it other than time, t, and you are to assume that the initial conditions of the system are zero.

(c) (10pts.) The forcing function is now changed to $\begin{Bmatrix}F_1(t)\\ F_2(t)\\ F_3(t)\end{Bmatrix}=\begin{Bmatrix}\alpha\\ \beta\\ \gamma\end{Bmatrix}\sin(\Omega t)$, where the constants $\alpha, \beta, \gamma \neq 0$. How many resonant peaks will there be in the frequency response functions for $|x_i(t)|$ for i=1,2,3? Briefly explain the reasoning behind your answer. (Here we define the i^{th} frequency response function to be the plot of the magnitude of the steady-state response of $x_i(t)$ as a function of the forcing frequency, Ω.)