Student Code Number:______________

Ph.D. Qualifying Exam

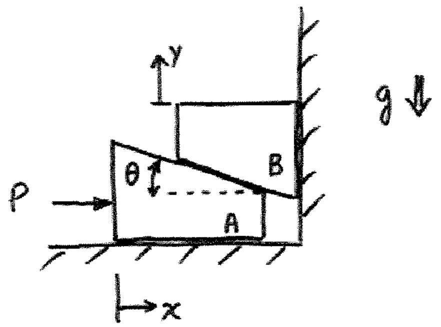
Dynamics and Vibrations

August 24, 2005

S. W. Shaw and B. Feeny

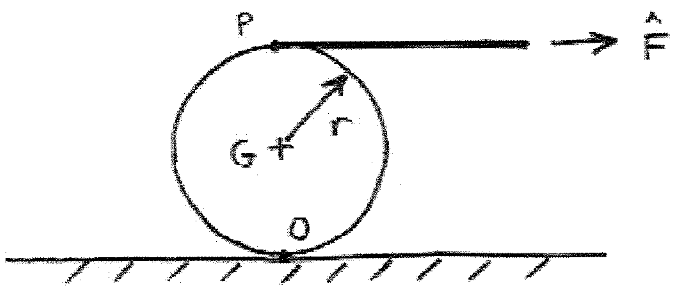
Directions: **Work all four problems.**
Note that the problems are EVENLY WEIGHTED.
You may use two books and two pages of notes for reference.

1. For the system below, as block A (of mass $m_A = 2$ kg) slides horizontally by an amount x, block B (of mass $m_B = 1$ kg) is constrained to slide vertically by an amount $y = x \sin \theta$. Given a horizontal force $P = 1$ N, determine the *acceleration vector* of block A. Neglect friction, and use $\theta = 30°$.



2. A spool of paper of radius r is shown at rest below. The mass of the spool is $m = \alpha r^2$, where $\alpha = 4000$ kg/m^2. For a spool of radius $r = 0.2$ m, a sudden short-time force is applied horizontally as shown, so as to rip the paper, resulting in rolling motion (no slip) at a speed of $v_G = 0.4$ m/s at the center of mass. This same sudden ripping action is applied to a spool with radius $r = 0.1$ m. Consider two models to find the resulting speed of its center of mass, assuming no slip.

(a) model the ripping action as an impulse $\hat{F} = F_{AVE}\Delta t$, which is the same for the cases of $r = 0.2$ m and $r = 0.1$ m.
(b) model the ripping action as an input of work $F_{AVE}\Delta x$, which is the same for the cases of $r = 0.2$ m and $r = 0.1$ m.
(c) comment on the difference between the models, and which you think is better.
(The mass moment of inertia of the spool is $I_G = \frac{1}{2}mr^2$, about the center of mass, where m is given above.)



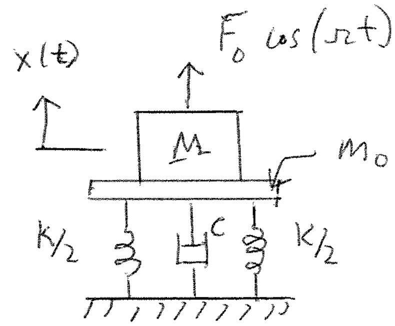
(this sketch is labeled for case (a) with $\hat{F} = F_{AVE}\Delta t$)

3. Consider an air-conditioning compressor that is to be placed on the roof of a building. The compressor has a total mass of M; it runs at a circular frequency (in *rad/sec*) of Ω and it has a small unbalance that leads to an effective applied force of the form $F_0 \cos(\Omega t)$. Here you will consider the design of a vibration isolation system, that is, the selection of a spring k, a dashpot c, and a mounting mass m_0, as shown below, in order to minimize the dynamic component of the force transmitted to the roof from the compressor vibrations (you do not need to consider static forces). Throughout the problem you are to use $r = \Omega / \omega_n$ as the frequency ratio parameter.

(a) For the first step of the design, you can assume zero damping and zero mounting mass, that is, $c = 0$ and $m_0 = 0$. Show that if one desires a value of α for the transmitted force ratio, F_T / F_0, the required support stiffness is given by $k = \dfrac{M(1+\alpha)}{\alpha \Omega^2}$. $(\alpha < 1)$.

(b) Now consider adding a mounting mass m_0 to the system with the design derived above. Does this result in a smaller or larger (dynamic) transmitted force? You must justify your answer.

(c) When the compressor is starting up and stopping it will pass through a resonance condition. If this happens slowly, vibrations can build up to unacceptable levels. Derive a relationship of the form $\zeta = f(\beta)$ between the damping ratio $\zeta = \dfrac{c}{2\sqrt{k(M+m_0)}}$ and the force transmitted near resonance, $\beta \cong F_T / F_0 \big|_{r=1}$. Note that a small damping assumption is used, resulting in β values larger than unity.



4. Consider the two-degree-of-freedom system shown below. The pendulum bar is assumed to be rigid and massless, and friction is neglected throughout the system. The mass, length, and stiffness coefficients shown are expressed in terms of nondimensional parameters, derived such that the gravitational constant g is unity.

(a) Show that the system mass and stiffness matrices are given by

$$\mathbf{M} = \begin{bmatrix} 1+\mu & 1 \\ 1 & 1 \end{bmatrix} \qquad \mathbf{K} = \begin{bmatrix} \kappa & 0 \\ 0 & 1 \end{bmatrix} \qquad \text{States} \begin{pmatrix} x \\ \theta \end{pmatrix}$$

(b) Show that the natural frequencies are given by the roots of the following equation:

$$\mu\omega^4 + (1+\kappa+\mu)\omega^2 + \kappa = 0$$

(c) For $\mu = 2$ and $\kappa = 2$ determine the system natural frequencies and mode shapes, AND determine the location of the node on the pendulum bar for the higher-frequency mode.

(d) Take μ and κ to be general and consider the system with the applied harmonic force as shown. Determine the frequency of excitation Ω such that in the resulting steady-state vibrations, the square mass remains stationary – that is, the pendulum acts as a tuned vibration absorber. (Note: steady-state is achieved due to very small dissipation effects that have been ignored.)