Student Code Number: __________

Ph.D. Qualifying Exam

Dynamics & Vibrations

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Directions: Work all four problems.
Note that the problems are EVENLY WEIGHTED.
You may use two books and two pages of notes for reference.

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1.

PART A:

A disc of radius R and mass m has a polar moment of inertia about its center, $J_O = \frac{1}{2}mR^2$. It is initially at rest on a slope of angle β as sketched below in Figure 1-A. You may assume there is no difference between kinetic and static friction and that the friction force, f, when slip occurs is modeled in the usual way as $f = \mu N$, where N is the normal reaction force between the body and the surface and μ is the coefficient of friction.

Derive the inequality between β and μ that will ensure that the disc will roll down the slope without slipping. Gravity is acting vertically downwards.

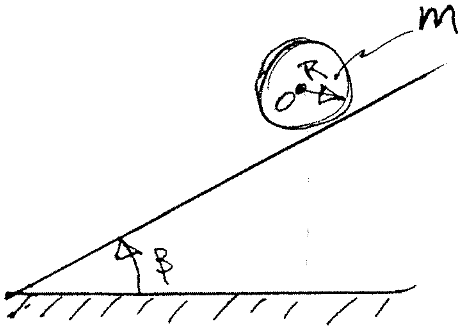

Figure 1-A

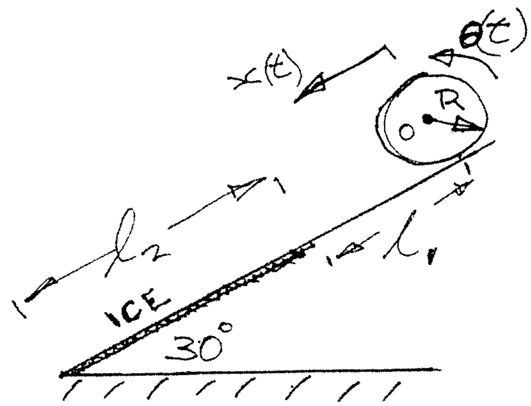

Figure 1-B

PART B:

There is a disc as in PART A that is held initially at rest on a slope. There is a coefficient of friction $\mu = 0.25$ at all points along the slope except for a region of length l_2, which is ice, where the coefficient of friction drops to zero. See the sketch Figure 1-B. Using the parameter values:

$l_1 = 1\text{ m}$

$l_2 = 2\text{ m}$

$R = 1\text{ m}$

$m = 1\text{ kg}$

$g = 10\text{ m/s}^2$

$\beta = 30^o$

(i) Derive expressions for the displacement of the center of the disc $x(t)$, measured parallel to the slope, and the angular position of the disc, $\theta(t)$. The disc starts from rest and gravity is acting vertically downward. Only consider motion while the disc is on the slope.

(ii) Sketch both of these coordinates as functions of time, taking care to show the correct form of the slopes of the functions before, at, and after the disc passes onto the ice.

2. A particle of mass m moves down a frictionless incline as shown in Figure 2. The particle is released (with zero velocity) from a height h above the bottom of the loop. Gravity acts vertically downward.

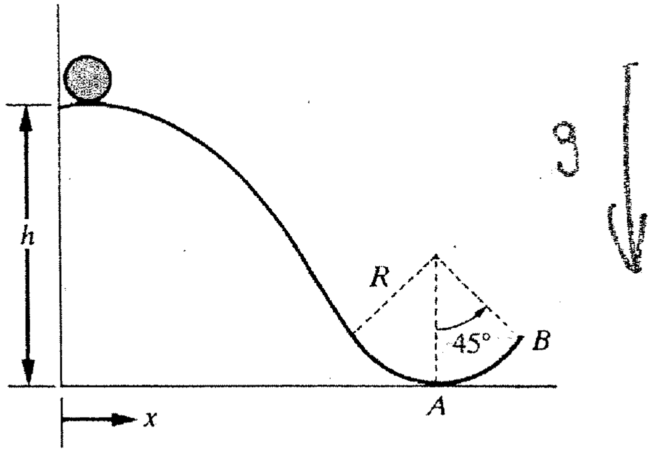

Figure 2

a. What is the force of the inclined track on the ball at the bottom (point A)?
b. At what velocity does the ball leave the track?
c. How far away from A does the ball land on the level ground?

3. The mass sketched in Figure 3-A has a slightly out of balance electric motor attached to it (not shown) that is rotating at 10 Hz. This motor creates a harmonic force $F(t) = 2\sin(2\pi 10t)$ N that acts on the mass as shown. The total mass of the system is $m = 2$ kg, the stiffness is $k = 8000$ N/m, and the damping coefficient is $c = 50$ Ns/m.

(a) Calculate the force transmitted to the ground after the electric motor has been spinning at 10Hz for a few minutes, i.e., all the transients have decayed to zero.

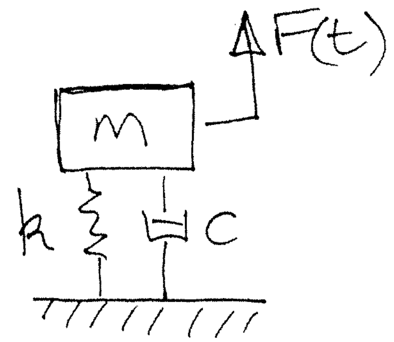

Figure 3-A

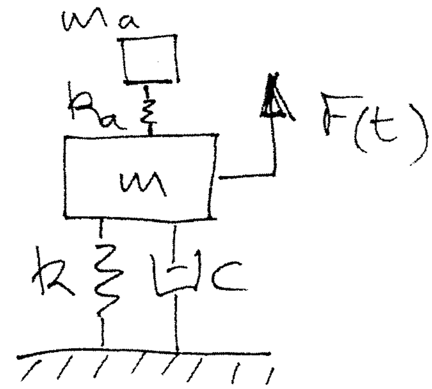

Figure 3-B

(b) A common way to reduce the vibration of a system is to add an absorber to it. In its simplest form, this is a mass m_a and a stiffness k_a attached to the original system as shown in Figure 3-B. Design an absorber that would reduce the original motion of the main mass, m. You may assume that the motor remains running at 10Hz. A rough estimate of the values of m_a and k_a will do providing that any simplifying assumptions you make are clearly stated.

(c) What other practical design changes could you make to the system of Figure 3-A so that the force transmitted to the ground is reduced? Assume that the motor must spin at 10Hz and that it cannot be balanced.

4. In the usual notation you are given that the equations of motion for a 3 DOF system are:

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&2\end{bmatrix}\begin{pmatrix}\ddot{x}_1\\\ddot{x}_2\\\ddot{x}_3\end{pmatrix}+\begin{bmatrix}.02&-.01&0\\-.01&.03&-.01\\0&-.01&.03\end{bmatrix}\begin{pmatrix}\dot{x}_1\\\dot{x}_2\\\dot{x}_3\end{pmatrix}+\begin{bmatrix}1&-1&0\\-1&2&-1\\0&-1&1\end{bmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}1\\1\\1\end{pmatrix}\sin(1.7t)$$

The coordinate transformation:

P

$\tilde{p}$

p

$$\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\underbrace{\begin{bmatrix}0.50&0.73&0.47\\0.50&0.20&-0.84\\0.50&-0.46&0.18\end{bmatrix}}_{E \cdot V}\begin{pmatrix}p_1\\p_2\\p_3\end{pmatrix}$$

$x_i/\sqrt{m_{ii}}$

$\omega_n - \frac{k}{m}$

is used to uncouple the equations into the form:

$$\begin{pmatrix}\ddot{p}_1\\\ddot{p}_2\\\ddot{p}_3\end{pmatrix}+\begin{bmatrix}0.01&0&0\\0&0.02&0\\0&0&0.04\end{bmatrix}\begin{pmatrix}\dot{p}_1\\\dot{p}_2\\\dot{p}_3\end{pmatrix}+\begin{bmatrix}0&0&0\\0&0.7&0\\0&0&2.8\end{bmatrix}\begin{pmatrix}p_1\\p_2\\p_3\end{pmatrix}=\begin{pmatrix}1.5\\f_2\\-0.2\end{pmatrix}\sin(1.7t)$$ ✓

$p = A(\omega_x t + d)$

Answer the following. You may utilize any or all of the numbers used above.

(i) Find the value of the coefficient f_2.

(ii) The 3[rd] mode shape i.e., the mode associated with $\omega_3^2 = 2.8$ is:

A. $\begin{pmatrix}.50\\-.46\\.18\end{pmatrix}$ B. $\begin{pmatrix}.73\\.20\\-.46\end{pmatrix}$ C. $\begin{pmatrix}.94\\-1.68\\.36\end{pmatrix}$ ✓ D. $\begin{pmatrix}-0.47\\-.84\\-.18\end{pmatrix}$

E. None of the above. EXPLAIN WHY.

(iii) Find the steady-state solution of $p_3(t)$.

3rd (iv) Explain which mode you would expect to dominate the response of the system.

(v) Describe the motion of the mode associated with the lowest of the three natural frequencies.

$[k - \omega^2 m]\{\hat{x}\} = 0$

$\det(\omega^2 I - \cancel{A}) = 0$

$\det(k - \omega^2 m) = 0$