Student Code Number:____________

Ph.D. Qualifying Exam

Dynamics and Vibrations

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Directions: **Work all four problems.**
Note that the problems are EVENLY WEIGHTED.
You may use two books and two pages of notes for reference.

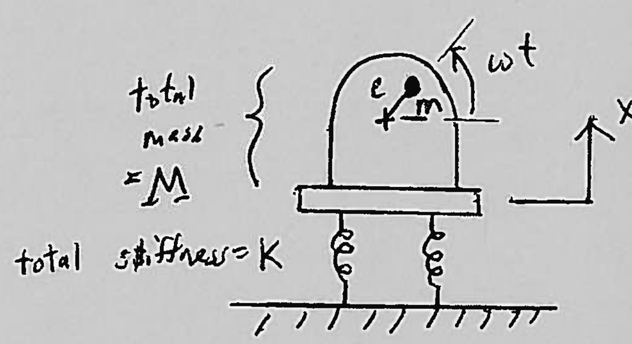
1. Consider the single degree of freedom system shown below, which is a model for a compressor sitting on the roof of a building. The compressor and its mounting plate have a total mass of M=10kg, and the compressor has a rotating mass imbalance ***me*** where ***m*** is the effective rotating mass and ***e*** is its eccentricity. The unit is sitting on a flexible mounting of total stiffness ***k*** such that it can move only vertically, and it has a measured natural frequency of ω_n=20 rad/sec, with very little damping. In the first trial run the compressor rotational speed is set at ω_1 =24 rad/sec, which is where it is to be operated in normal steady state operation. At this operating point the magnitude of the steady-state response of the compressor is measured to be A=4mm, where $x_{SS} = A\sin(\omega t - \phi)$, and the magnitude of the dynamic force (that is, its oscillating component) transmitted to the roof is excessive and must be reduced. Answer the following questions, clearly stating any assumptions you make, and giving specific technical reasons for your answers. Note that the coordinate x shown in the diagram below is measured such that x=0 is the system static equilibrium position.

(i) Compute the rotating imbalance quantity ***me***, in units of kg-m.

(ii) Compute the amplitude of the harmonic force F_T that is being transmitted to the roof in the trial run, expressed in Newtons.

(iii) If the transmitted force is to be reduced, should one decrease or increase the mass of the mounting? Be very specific about the reasons for your answer.

(iv) If one wanted to reduce the transmitted force by a factor of 3, by what factor should M be increased, that is, what should be the value of γ if the new total mass is to be $M_2=\gamma M$? Note that γ>1.



2. The overall rotation and first torsional vibration mode of a crankshaft can be modeled by the two degree of freedom system depicted below. Note that the torque converted is modeled by a viscous coupler to ground at one end with coefficient c, that is, it produces a torque of magnitude $c\dot{\theta}_2$ acting on inertia I_2. Note that $I_1=I$ and $I_2=2I$, and also note that the only element acting between I_2 and ground is the viscous coupler.

(i) Show that the system mass, damping, and stiffness matrices are as follows:

$$\mathbf{M}=\begin{pmatrix} I & 0 \\ 0 & 2I \end{pmatrix} \quad \mathbf{C}=\begin{pmatrix} 0 & 0 \\ 0 & c \end{pmatrix} \quad \mathbf{K}=\begin{pmatrix} k & -k \\ -k & k \end{pmatrix}$$

(ii) Determine whether or not the system has proportional (Rayleigh) damping, that is, whether or not there exist coefficients α and β such that $\mathbf{C}=\alpha\mathbf{M}+\beta\mathbf{K}$.

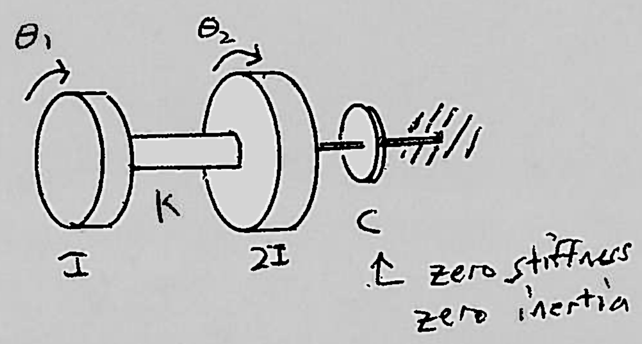
(iii) Determine the system's **undamped** ($c=0$) natural frequencies and mode shapes, and comment on the physical nature of the modes and any special features they may possess.

(iv) Ignore damping ($c=0$) again for this part: Consider an applied harmonic torque $T=T_1\sin(\omega t)$ applied to inertia I_1. Sketch the frequency response curves for $|\theta_1|$ and $|\theta_2|$ versus ω for this excitation, noting all important features on the curves, including resonances, zeros, and the limits as ω goes to zero and to infinity.

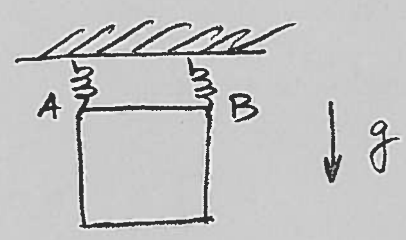
(v) Again, consider the excitation from part (iv). Use the results from part (iv), or some other method, to determine the frequency at which the steady-state amplitude of vibration of I_1 is zero under the harmonic excitation, and comment on how this can occur *in light of the fact that the applied torque is acting directly on* I_1. Use physical arguments as well as mathematics in your answer.

(vi) Include the torque converter damping in this part ($c \neq 0$): If a constant torque T_0 is applied to inertia I_1, (with no oscillating torque), determine the resulting steady-state crankshaft speed Ω when the entire rotor is running at a constant speed of rotation Ω. Express your result in terms of T_0, c, and k (or a subset of these parameters).

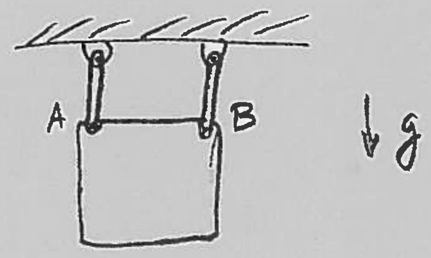
(vii) For the same conditions as part (vi), determine the steady-state crankshaft wind-up, that is, the relative angle between I_1 and I_2. Express your result in terms of T_0, c, and k (or a subset of these parameters).



3. (i) A thin square uniform plate of mass m is initially held in place by two identical springs as shown. If the spring at point B suddenly breaks, find the angular acceleration of the plate and the force in the intact spring at the instant after release.



(ii) Redo the problem for the case where the same plate is initially held in place by two identical massless rigid links such that the link at point B is suddenly released. Again find the angular acceleration of the plate and the force in the remaining link at the instant after release.



(iii) State which of the above two problems is more difficult, and give reasons for your answer.

4. (i) A pellet of mass M is released from rest and slides down the frictionless linear ramp from point A to point B as shown in the top figure. How much time does it take?

(ii) The same pellet will now slide from point A to point B along a two part path, the first part is a linear ramp and the second part is a horizontal segment as shown in the bottom figure. The two parts meet at point C where $x = \xi L$. The pellet is again released from rest at point A and there is again no friction. The transition at C is locally smoothed so that there is no energy loss. Find the value of ξ that makes for the shortest time of travel.

(iii) Consider the situation in part (ii) but now suppose that the pellet is replaced by a uniform disk that rolls along the path. Will the value of ξ increase or decrease? You can answer by doing detailed calculations, or you can provide a qualitative answer that is supported by principles of mechanics.