Student Code Number: 4

Ph.D. Qualifying Exam

Dynamics and Vibrations

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Directions: Work all four problems.
Note that the problems are EVENLY WEIGHTED.
You may use two books and two pages of notes for reference.

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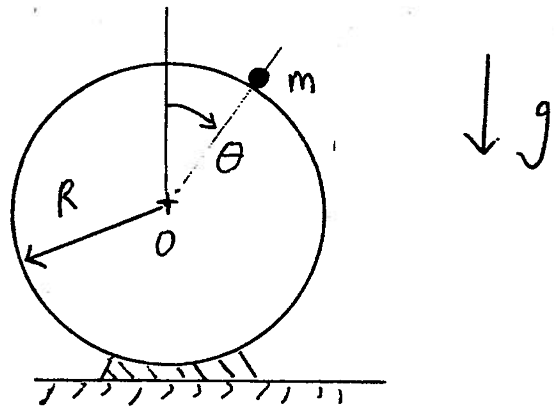
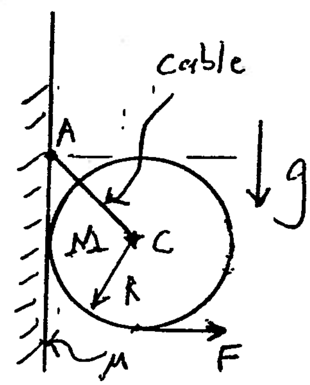
1. A mass particle m is sitting atop a smooth (frictionless) sphere of radius R that is securely fastened to the table as shown. It is nudged slightly and begins to slide down the surface of the cylinder under the action of gravity. Determine the angle θ^* at which the particle will leave the surface of the cylinder.



2. A uniform cylinder of mass M and radius R rests against a rough wall and is held in place by an inextensible massless cable. The cylinder is at rest, and then a horizontal force F is suddenly applied, as shown. The coefficient of sliding (Coulomb) friction between the cylinder and the wall is given by μ. Determine the angular acceleration α_0 of the cylinder and the tension T_0 in the cable at the instant immediately after F is applied. You may assume that F is in a range such that the cylinder begins to rotate, yet it remains in contact with the wall.



3. A 1 DOF system sketched below is acted on by a sinusoidal force and the resulting steady-state motion is given by $x(t) = X_1 \sin(\omega t - \phi_1)$. The magnitude of the steady-state response, X_1 as a function of the forcing frequency, is plotted in Figure 3a.

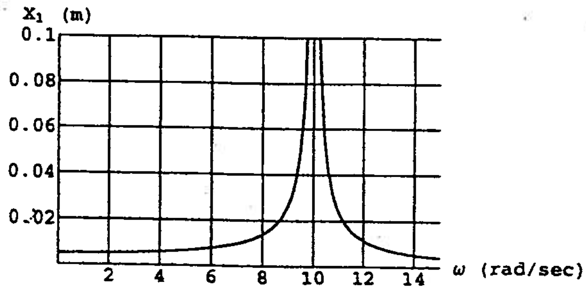
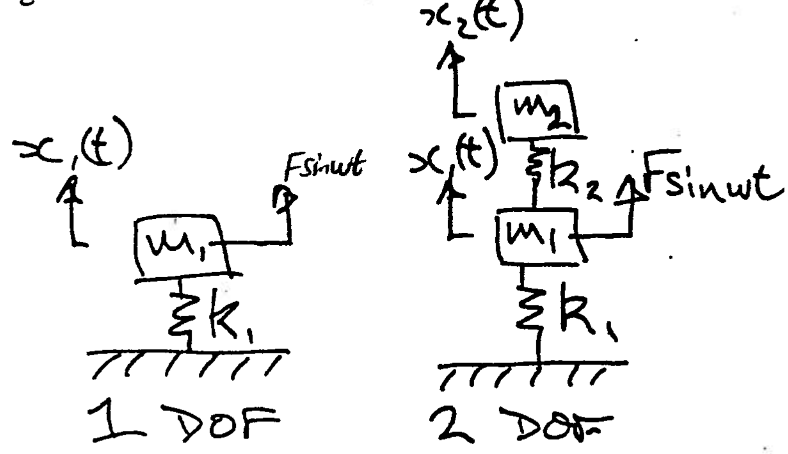


Figure 3a.



A second mass is added to the first to create a 2 DOF as sketched above. The associated magnitudes of the steady-state responses X_1 and X_2 are plotted below in Figures 3b and 3c, repectively.

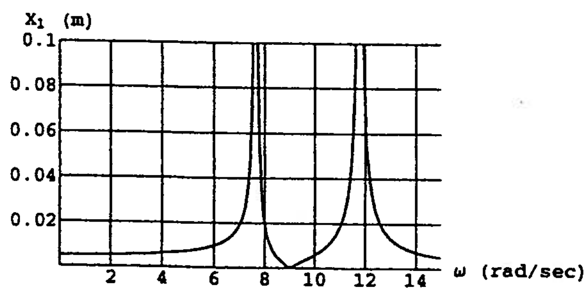


Figure 3b.

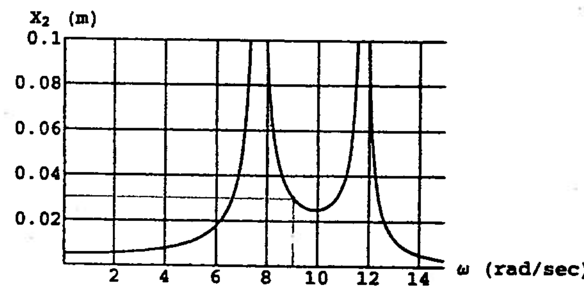


Figure 3c.

(i) With your knowledge of vibration absorbers (or otherwise), what is the value of the ratio $\sqrt{k_2/m_2}$? Why?

(ii) Estimate, with justification, the magnitude of the applied force, F.

(iii) For this part of the question, assume the forcing frequency, ω, is 8 rad/sec. Has the magnitude of the motion of mass 1, i.e., X_1, been increased or decreased by the addition of the absorber mass 2? Clearly explain your answer by making use of Figures 3a, b, and/or c, or otherwise.

4. In the usual notation you are given that the equations of motion for a 3 DOF system are:

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 2\end{bmatrix}\begin{pmatrix}\ddot{x}_1\\\ddot{x}_2\\\ddot{x}_3\end{pmatrix}+\begin{bmatrix}.05 & -.05 & 0\\-.05 & .1 & -.05\\0 & -.05 & .05\end{bmatrix}\begin{pmatrix}\dot{x}_1\\\dot{x}_2\\\dot{x}_3\end{pmatrix}+\begin{bmatrix}1 & -1 & 0\\-1 & 2 & -1\\0 & -1 & 1\end{bmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}1\\1\\0\end{pmatrix}\sin(1.7t)$$

The coordinate transformation:

$$\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{bmatrix}0.50 & 0.7257 & -0.4727\\0.50 & 0.2037 & 0.8417\\0.50 & -0.4647 & -0.1845\end{bmatrix}\begin{pmatrix}p_1\\p_2\\p_3\end{pmatrix}$$

is used to uncouple the equations into the form:

$$\begin{pmatrix}\ddot{p}_1\\\ddot{p}_2\\\ddot{p}_3\end{pmatrix}+\begin{bmatrix}0 & 0 & 0\\0 & 0.036 & 0\\0 & 0 & 0.139\end{bmatrix}\begin{pmatrix}\dot{p}_1\\\dot{p}_2\\\dot{p}_3\end{pmatrix}+\begin{bmatrix}0 & 0 & 0\\0 & 0.7192 & 0\\0 & 0 & 2.7808\end{bmatrix}\begin{pmatrix}p_1\\p_2\\p_3\end{pmatrix}=\begin{pmatrix}1.0000\\f_2\\-0.3690\end{pmatrix}\sin(1.7t)$$

Answer the following:

(i) What is the value of the coefficient f_2?

(ii) The 3rd mode shape, i.e., the mode associated with $\omega_3^2 = 2.7808$ is very close to: (circle the correct response and EXPLAIN WHY you have chosen this answer)

A. $\begin{pmatrix}0.50\\-0.46\\-0.18\end{pmatrix}$ B. $\begin{pmatrix}0.95\\-1.68\\0.37\end{pmatrix}$ C. $\begin{pmatrix}0.50\\-0.73\\0.47\end{pmatrix}$ D. $\begin{pmatrix}-0.47\\-.84\\-.18\end{pmatrix}$ E. *None of the answers*

(iii) Find the steady-state solution (i.e., particular solution) of $p_1(t)$, clearly explaining the physical interpretation of your answer in terms of that coordinate's natural frequency.

(iv) One mode is likely to dominate the response of the system. Which mode do you think this is and why? What would be the associated one-mode approximation to the steady-state responses $x_1(t)$, $x_2(t)$, and $x_3(t)$?