Student Code Number:____________

Ph.D. Qualifying Exam

Dynamics and Vibrations

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Directions: Work all four problems.
Note that the problems are EVENLY WEIGHTED.
You may use two books and two pages of notes for reference.

1. You are going to borrow a trailer built by your brother-in-law to haul some building supplies. It seems to bounce a lot, so you run a test by pushing down on the empty trailer and releasing it, and you measure that it takes 3 seconds for five complete oscillation cycles, during which time the amplitude reduces by about 50%, that is, it reduces by about 13% each cycle. You load 400 pounds (180 kg) of materials onto the trailer, which causes a deflection of 3 inches (7.6 cm) of the trailer.

a) Determine the damping ratio and the undamped natural frequency of the **empty** trailer, and state whether or not a small damping assumption is valid in your calculations.
b) Determine the mass of the trailer (in kg), the stiffness of the suspension springs (in N/m), and the viscous damping coefficient (in kg/sec) of the shock absorbers.
c) Determine the damping ratio and the undamped natural frequency of the **loaded** trailer, and state your assumptions.
d) If you hit a bump with the loaded trailer, how long will it take for resulting oscillations to die out, say, to within 10% of their highest amplitude?
e) If you drive over a bridge with bumps spaced at regular intervals of 15m, explain why the loaded trailer bounces violently at speeds of approximately 18m/s (40mph) and 9m/s (20mph), but not at any other speeds in the range 20-60mph.
f) If you replace the shock absorbers with new ones, and you want the loaded trailer oscillations to die out as fast as possible, what value of viscous damping coefficient should they have (in kg/sec)?

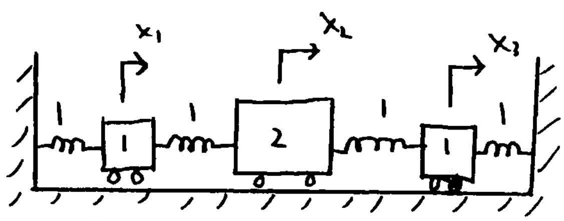
2. Consider the three degree of freedom system shown below. Note that some system data, including the mass and stiffness matrices, and two natural frequencies and two mode shapes, are provided (in some consistent set of units).

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{K} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

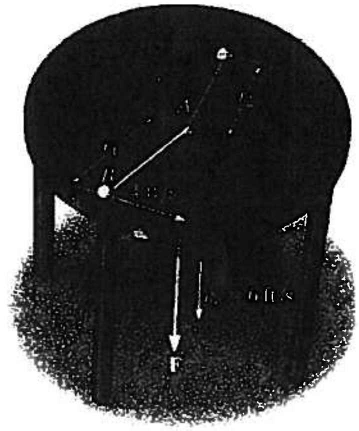
$\omega_1 = 0.618$ $\qquad$ $\omega_2 = ???$ $\qquad$ $\omega_3 = 1.618$

$$\mathbf{u_1} = \begin{pmatrix} 1 \\ 1.618 \\ 1 \end{pmatrix} \qquad \mathbf{u_2} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \qquad \mathbf{u_3} = \begin{pmatrix} 1 \\ ?? \\ ?? \end{pmatrix}$$

a) Determine the unknown elements in the third mode shape.
b) Determine the second natural frequency.
c) Determine the response of the system after it is released with the following set of initial conditions: $\mathbf{x}(0) = (0, 1, 0)$ and $\dot{\mathbf{x}}(0) = (0, 0, 0)$ AND comment on any special features of this response. This initial state is simply the central mass displaced while the outer masses are held at equilibrium, and the system released from rest.
d) Consider the case in which the leftmost mass is forced with a harmonic force $f_1 \sin(\omega t)$, and the system response is measured by an accelerometer placed on the central mass. Determine the number of resonance peaks this accelerometer will capture for the steady-state response as the frequency is varied. Give sound reasons, or provide detailed calculations, for your answer.
e) Consider a harmonic force $f_2 \sin(\omega t)$ acting on the central mass. Determine the value of the excitation frequency ω such that the central mass does move in steady-state operation, and comment on how this is physically possible.



3. The 0.8-lb ball B, is attached to a cord of negligible mass which passes through a hole at A in the smooth table. When the ball is $r_1 = 1.75$ ft from the hole, it is rotating around in a circle such that its speed is $v_1 = 4$ ft/sec. By applying the force $\boldsymbol{F}$ the cord is pulled downward through the hole with a constant speed $v_c = 6$ ft/sec. Find (a) the speed of the ball at the instant it is $r_2 = 0.6$ ft from the hole, (b) the force $\boldsymbol{F}$ at this same instant, and (c) the amount of work done by the force in shortening the radial distance on the table from r_1 to r_2. Neglect the size of the ball.



4. The 10-kg wheel has a moment of inertia $I_G = 0.156$ kg m^2. Assuming that the wheel does not slip or rebound, determine the minimum velocity v_G it must have to just roll over the obstruction at A.