**Student Code Number:____________**

**Ph.D. Qualifying Exam**

# Dynamics and Vibrations

**August, 2008**

**S. Shaw and A. Haddow**

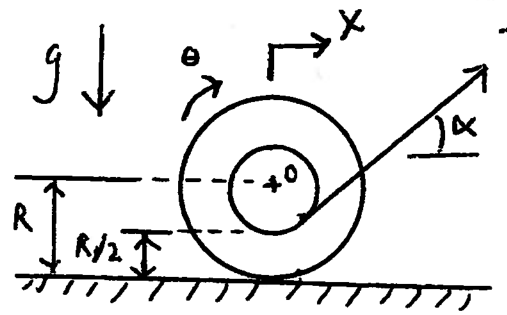
**<u>Directions</u>:** **Work all four problems.**
**Note that the problems are EVENLY WEIGHTED.**
**You may use two books and two pages of notes for reference.**

Rev. 5/14/08

1. The system shown consists of a stepped cylinder of large radius *R*, small radius *R*/2, mass *m*, and moment of inertia $I_O = mR^2/3$ about the center of mass *O*. A cable is wrapped around the smaller radius and is being pulled with a tension *T* at an angle $\alpha$, as shown.

(a) Assuming the cylinder rolls without slipping, determine its acceleration $\ddot{x}$. Express your results in terms of parameters $T, m, R, \alpha, g$.
(b) Derive and discuss the conditions under which the cylinder rolls to the left or to the right, under the no-slip assumption.
(c) Consider a coefficient of friction $\mu$ between the cylinder and ground (assuming that the static and kinetic friction coefficients are equal). Determine the maximum value of the tension *T* for which slip does not occur. Express your results in terms of parameters $m, R, \alpha, g, \mu$.

$0 \leq \alpha < \pi/2$

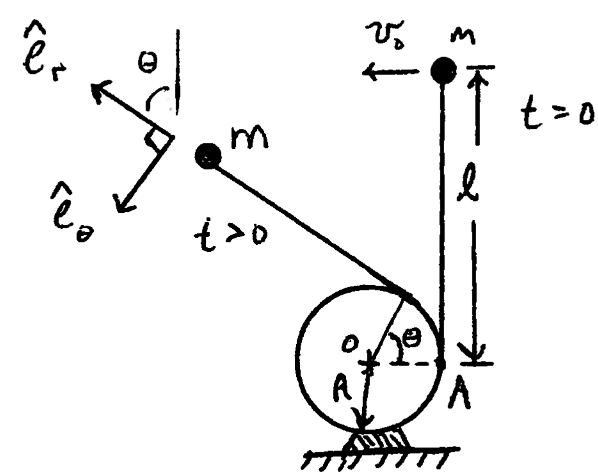


2. A mass $m$ is attached to one end of an inextensible, massless string of total length $\ell$. The other end of the string is attached (at point $A$ in the diagram below) to a fixed, nonrotating cylinder of radius $R$. The system starts with the string fully extended in the purely tangential configuration shown below (at $t=0$), with the mass moving with a velocity of magnitude $v_0$, as indicated. As the string wraps on the cylinder, the system configuration is dictated by the angle $\theta$. All motion occurs in a horizontal plane, so that gravity can be ignored. Note that an important key to solving this problem correctly is to carefully write the position vector of the mass, since this is used as the starting point for your calculations. Also note that the unit vectors $\hat{e}_r$ and $\hat{e}_\theta$ depicted in the diagram are very useful for solving this problem; they are defined with respect to fixed axes by the same angle $\theta$ that is used for the system coordinate, as indicated.

(a) Show that the total energy of the system is given by $E = \frac{m}{2}\dot{\theta}^2(\ell - R\theta)^2$, and state why it is conserved during the motion.

(b) Determine the tension $T$ in the string as a function of $\theta$ and the parameters $m, \ell, R, v_0$.

(c) Determine the angular acceleration $\ddot{\theta}$ as a function of $\theta$ and the parameters $m, \ell, R, v_0$.



3. This problem deals with a car traveling along a sinusoidally undulating road. The car moves at a constant speed, $v$, in the horizontal direction and it may be considered to act as a one degree of freedom system in the vertical direction. The mass of the car is $m$, its suspension stiffness and damping coefficient are $k$ and $c$ respectively. The undulations in the road have a wavelength of $\lambda$ and a peak-to-peak amplitude of $2A$. The wavelength, $\lambda$, is large compared to the length of the car and the damping, $c$, is small. See the figure below.

In answering the following questions, you may make any reasonable assumptions that have a negligible affect on the answers. State any such assumptions you make.

(i) In terms of the other parameters in the problem, what speed, $v$, would cause the largest vertical motion of the car?
(ii) What speed should the car have to assure that its absolute vertical motion always remains less than 10% of the amplitude of the road undulation, $A$?
(iii) What is the maximum force exerted on the road by the car when the car is moving extremely fast?
(iv) Clearly explain how you would check that the tire of the car was remaining in contact with the road. You do NOT need to solve for this condition, but carefully present the conditions(s) that have to be considered.

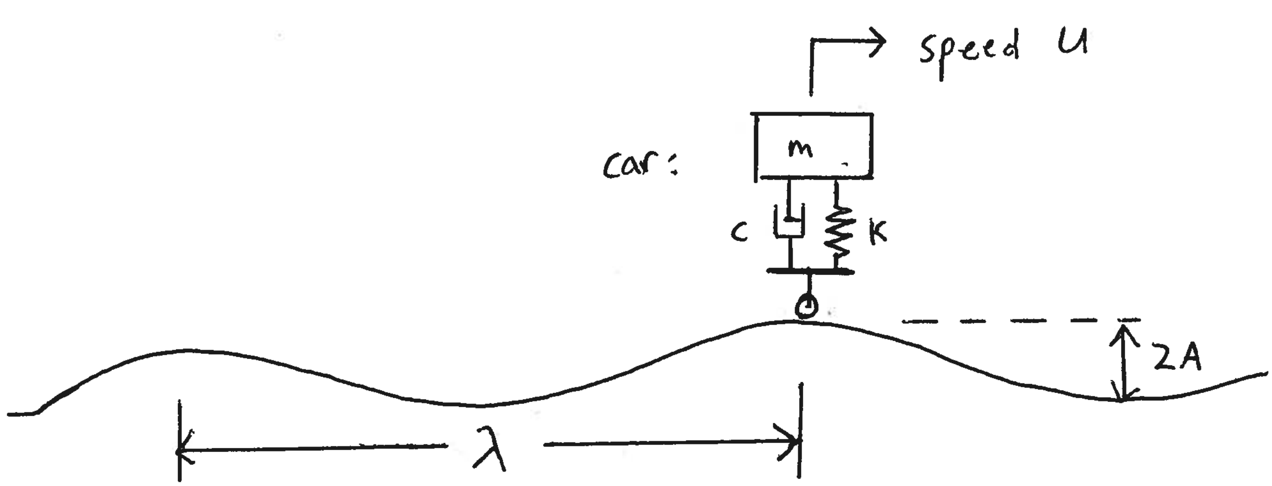



Figure 3

4. The equations of motion of a 3DOF system are:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}\mathbf{x}+\begin{pmatrix} 0.08 & 0 & 0 \\ 0 & 0.16 & 0 \\ 0 & 0 & 0.08 \end{pmatrix}\mathbf{x}+\begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix}\mathbf{x}=\begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix}\sin\Omega t$$

The transformation $\mathbf{x}=\mathbf{P}\mathbf{p}$, where $\mathbf{P}=\begin{pmatrix} 0.5 & -0.707 & -0.5 \\ 0.5 & 0.000 & 0.5 \\ 0.5 & 0.707 & -0.5 \end{pmatrix}$, uncouples the equations as follows,

$$\mathbf{p}+0.24\mathbf{p}+\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}\mathbf{p}=\begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix}\sin\Omega t$$

(a) Physical describe the motion of each of the 3 modes and clearly explain if any oscillation could occur when the system is unforced (i.e., $\boldsymbol{F}=\mathbf{0}$)

(b) Given the initial conditions, $\mathbf{x}(0)=\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ and $\mathbf{x}(0)=\mathbf{0}$, find the homogenous solutions for $x_i(t)$, $i$=1, 2, and 3. (i.e., the solution with $\boldsymbol{F}=\mathbf{0}$)

(c) If $\begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix}=\begin{pmatrix} 1 \\ \alpha \\ \beta \end{pmatrix}$, find values for $\alpha$ and $\beta$ such that the third mode associated with a natural frequency of $\sqrt{2}$ will not be excited.