Student Code Number:____________

Ph.D. Qualifying Exam

Dynamics and Vibrations

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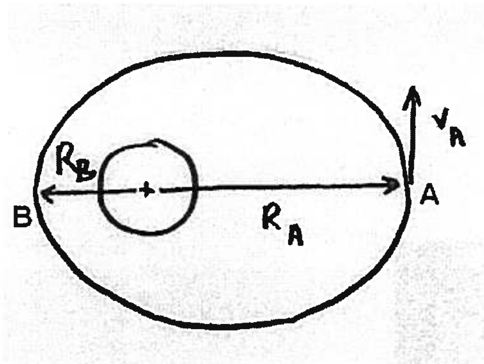
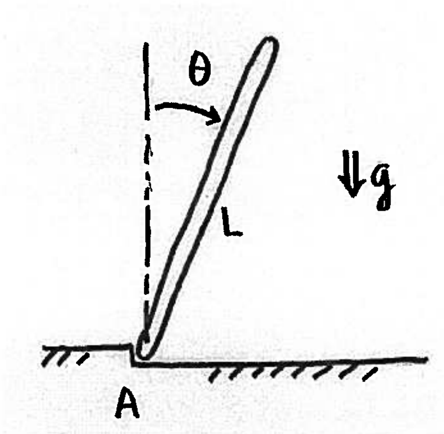
Directions: Work all four problems.
Note that the problems are EVENLY WEIGHTED.
You may use two books and two pages of notes for reference.

1. A starship of mass m is going to orbit about a strange alien planet of mass M. The captain of the starship wants to enter at the point A at a maximum distance $R_A = 12000$ km, and achieve a free orbit with a minimum distance at B of $R_B = 6000$ km. Determine the starship's speed v_A at the position A of the orbit, and speed v_B at the position B of the orbit. The mass of the planet is $M = 6 \times 10^{24}$ kg. Use a gravitational constant of $G = 66.7 \times 10^{-12}$ m^3/kg/s^2. (Treat the masses as $M >> m$, such that the planet is stationary while the starship is in orbit. Use general concepts of dynamics to solve this problem.)



2. A uniform bar of length L and mass m is in a vertical position, and due to a slight disturbance, falls toward the right at an increasing angle θ. Its lower end (point A) in contact with a nook in the ground, preventing the lower end of the bar from sliding. Write an expression as a function of angle θ, indicating at what angle the end of the bar loses contact at A. (Neglect friction at the point A.) Use $I_G = \frac{1}{12}mL^2$ for the bar.



3. As illustrated in Figure 3, a vibration isolation block is to be installed in a laboratory so that the vibration from an adjacent factory operation will not disturb certain experiments. If the mass of the isolation block is 500 kg and the surrounding floor and foundation vibrate at 30Hz with an amplitude of 0.3mm, determine the stiffness k of the isolation system such that the isolation block will have a steady-state amplitude 5 times smaller than the surrounding floor. State any assumptions

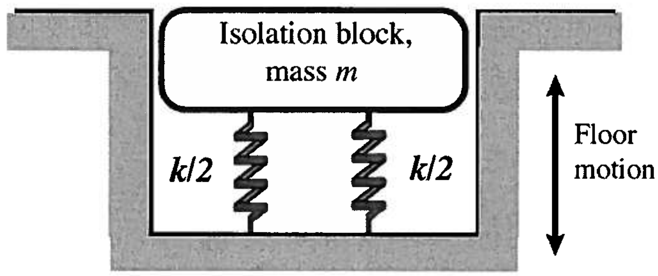


Figure 3

4. The system shown in Figure 4 has the following equation of motion in $\mathbf{w}$ coordinates:

$$m\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 1 & 1 & -1 \end{bmatrix}\begin{pmatrix} \ddot{w}_1 \\ \ddot{w}_2 \\ \ddot{w}_3 \end{pmatrix} + k\begin{bmatrix} 2 & 0 & 0 \\ -4 & 0 & -2 \\ 2 & 0 & -2 \end{bmatrix}\begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

where $w_2 \equiv x_2$ and where the relative displacements $w_1 = x_1 - x_2$ and $w_3 = x_1 - x_3$ are used.

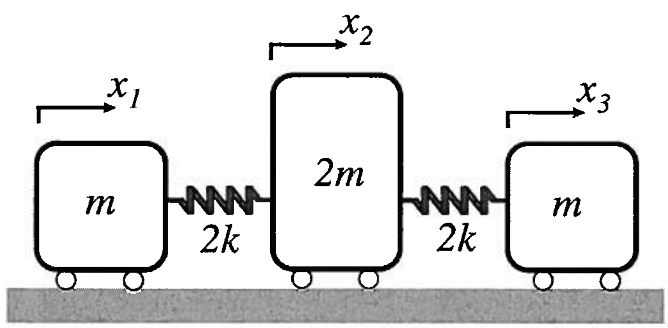


Figure 4

Answer the following:

(a) Determine the natural frequencies of the system

(b) Determine the mode shapes (eigenvectors) of the system, expressed in terms of the $\mathbf{w}$ coordinates and scaled so that the maximum entry in each of the vectors is 1.

(c) Sketch the three mode shapes in absolute (i.e., $\mathbf{x}$ coordinates)

(d) If the initial velocity of each mass is zero and the initial displacements are such that the modal displacement in the first (i.e., the lowest natural frequency) mode is non-zero, describe the resulting motion. How does the potential energy vary in this mode? Explain.