

Code No. _____

Thermodynamics Ph.D Qualifier Exam

Department of Mechanical Engineering
Michigan State University

August 2020

Directions: Closed book. A formula sheet is provided.

All problems carry equal weight. In order to receive full credit for a solution, you must show all work clearly.

Exam prepared by Profs. Brereton and Wichman

Problem 1.

Air flows into a well-insulated compressor at 200 kPa, 25°C and 65 m/s and exits at 4 MPa and 200 m/s. If the mass flow rate through the compressor is 45 kg/s and the required power input of the equivalent isentropic compressor is 85% of this one, find the power input and the temperature of air at outflow.

Assume air is an ideal gas with $R = 0.287 \text{ kJ/kg/K}$, $c_{p_0} = 1.0035 \text{ kJ/kg/K}$, $c_{v_0} = 0.7165 \text{ kJ/kg/K}$.

Problem 2.

A thermodynamic device is to be designed to use energy transfers from a heat source at 200°C and the ambient at 20°C to produce refrigeration at -30°C, with no external power input. What would be the ratio of heat transfer from the 200°C source to heat transfer from the -30°C cold space if the device were a Carnot device?

Final Formula Sheet

Property Relationships

Ideal gas equation of state:

$$p = \rho RT , \quad pV = RT , \quad pV = mRT , \quad pV = n\bar{R}T$$

$$u_2 - u_1 = c_v (T_2 - T_1) , \quad h_2 - h_1 = c_p (T_2 - T_1)$$

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} , \quad s_2 - s_1 = c_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$$

The quality x :

$$x = \frac{m_{vap}}{m_{vap} + m_{liq}}$$

$$v_{ave} = v_f + x(v_g - v_f) , \quad u_{ave} = u_f + x(u_g - u_f) , \quad h_{ave} = h_f + x(h_g - h_f) , \quad s_{ave} = s_f + x(s_g - s_f)$$

Work Relationships

Quasi-equilibrium work (any process):

$$_1W_2 = \int_1^2 p dV$$

Polytropic processes: $pV^n = \text{constant}$, $_1W_2 = \int_1^2 p dV = \frac{1}{1-n} [p_2V_2 - p_1V_1]$ when $n \neq 1$

First Law for a System

$$_1Q_2 - _1W_2 = E_2 - E_1 , \quad \dot{Q} - \dot{W} = \frac{dE}{dt} \quad \text{where} \quad E = m \left(u + \frac{1}{2}V^2 + gz \right)$$

First Law for a Control Volume

$$_1Q_2 - _1W_2 + _1m_{in2} \left(h_{in} + \frac{1}{2}V_{in}^2 + gz_{in} \right) - _1m_{out2} \left(h_{out} + \frac{1}{2}V_{out}^2 + gz_{out} \right) = E_2 - E_1$$

$$\dot{Q} - \dot{W} + \dot{m}_{in} \left(h_{in} + \frac{1}{2}V_{in}^2 + gz_{in} \right) - \dot{m}_{out} \left(h_{out} + \frac{1}{2}V_{out}^2 + gz_{out} \right) = \frac{dE}{dt}$$

Second Law for a System

$$_1P_{S2} + \int_1^2 \frac{\delta Q}{T_Q} = S_2 - S_1 , \quad \dot{P}_S + \frac{\dot{Q}}{T_Q} = \frac{dS}{dt}$$

Second Law for a Control Volume

$$_1P_{S2} + \int_1^2 \frac{\delta Q}{T_Q} + _1m_{in2}s_{in} - _1m_{out2}s_{out} = S_2 - S_1 , \quad \dot{P}_S + \frac{\dot{Q}}{T_Q} + \dot{m}_{in}s_{in} - \dot{m}_{out}s_{out} = \frac{dS}{dt}$$

Pr. 3

Boil water: One kilogram of liquid water at $T_l = 20^\circ\text{C}$ is placed into an open kettle. Heat is added at a fixed rate. The pressure is *approximately one atmosphere* throughout the process.

- (a) **(2/3 credit)** It takes 2 minutes to raise T to $\sim 100^\circ\text{C}$ ($x = 0$). How long does it take to vaporize the water completely ($x = 1$)? Use $c = 4.184 \text{ kJ/kg}\cdot\text{K}$ as the specific heat of liquid water.
- (b) **(1/3 credit)** Draw the process on p - v diagram, labeling all points including point l where the process starts (i.e., where $T = 20^\circ\text{C}$).

Thermodynamic Properties of Steam

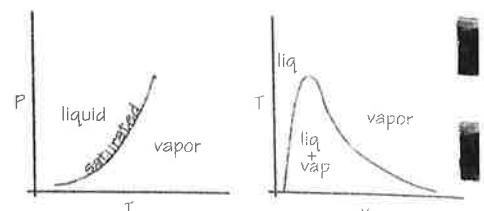


Table C.2: Saturated Steam Pressure Table

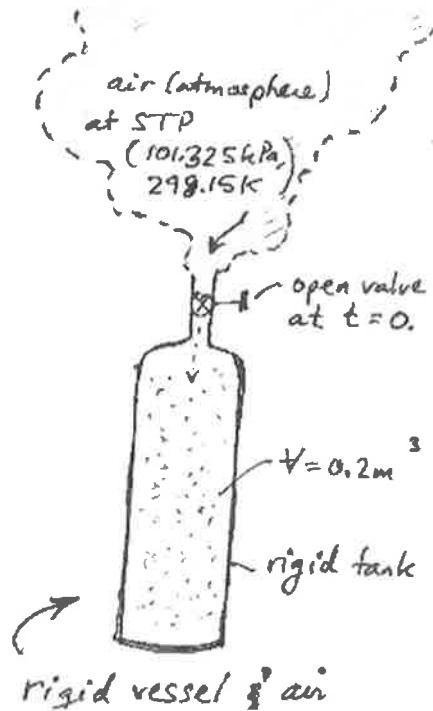
p (kPa)	T_{sat} ($^{\circ}\text{C}$)	Specific volume m^3/kg		Specific internal energy kJ/kg			Specific enthalpy kJ/kg			Specific entropy kJ/kg/K			
		Sat.		Sat.			Sat.			Sat.			
		liq.	v _f	liq.	u _f	u _{fg}	u _g	h _f	h _{fg}	h _g	s _f	s _{fg}	s _g
6113	0.01	0.001000	206.15345	0.00	2375.3	2375.3	0.01	2501.3	2501.3	0.0000	9.1562	9.1562	
1.0	6.98	0.001000	129.20803	29.29	2355.7	2385.0	29.29	2484.9	2514.2	1.059	8.8697	8.9756	
1.5	13.03	0.001001	87.98013	54.70	2338.6	2393.3	54.70	2470.6	2525.3	1.1956	8.6322	8.8278	
2.0	17.50	0.001001	67.00385	73.47	2326.0	2399.5	73.47	2460.0	2533.5	2.2607	8.4629	8.7236	
2.5	21.08	0.001002	54.25385	88.47	2315.9	2404.4	88.47	2451.6	2540.0	3.120	8.3311	8.6431	
3.0	24.08	0.001003	45.66502	101.03	2307.5	2408.5	101.03	2444.5	2545.5	3.3545	8.2231	8.5775	
3.5	26.68	0.001003	39.47651	111.87	2300.2	2412.0	111.88	2438.3	2550.2	3.3908	8.1315	8.5223	
4.0	28.96	0.001004	34.80015	121.44	2293.7	2415.2	121.44	2432.9	2554.4	4.4226	8.0520	8.4746	
4.5	31.02	0.001005	31.13887	130.01	2287.9	2418.0	130.01	2428.1	2558.1	4.4508	7.9817	8.4326	
5.0	32.88	0.001005	28.19251	137.79	2282.7	2420.5	137.79	2423.7	2561.4	4.4763	7.9187	8.3950	
6.0	36.16	0.001006	23.73939	151.51	2273.4	2424.9	151.51	2415.9	2567.4	5.5209	7.8094	8.3303	
7.0	39.00	0.001007	20.52957	163.37	2265.4	2428.8	163.38	2409.1	2572.5	5.5591	7.7167	8.2758	
8.0	41.51	0.001008	18.10327	173.85	2258.3	2432.1	173.86	2403.1	2577.0	5.5925	7.6361	8.2286	
9.0	43.76	0.001009	16.20309	183.25	2251.9	2435.2	183.26	2397.7	2581.0	6.6223	7.5648	8.1872	
10	45.81	0.001010	14.67355	191.79	2246.1	2437.9	191.81	2392.8	2584.6	6.6492	7.5010	8.1301	
12	49.42	0.001012	12.36103	206.88	2235.8	2442.7	206.90	2384.1	2591.0	6.6962	7.3900	8.0863	
14	52.55	0.001013	10.69348	219.95	2226.9	2446.8	219.97	2376.6	2596.6	7.7366	7.2959	8.0324	
16	55.32	0.001015	9.43255	231.52	2219.0	2450.5	231.53	2369.9	2601.4	7.7719	7.2140	7.9859	
18	57.80	0.001016	8.44473	241.90	2211.9	2453.8	241.92	2363.8	2605.8	8.8034	7.1416	7.9450	
20	60.06	0.001017	7.64937	251.35	2205.4	2456.7	251.38	2358.3	2609.7	8.8319	7.0766	7.9085	
25	64.97	0.001020	6.20424	271.88	2191.2	2463.1	271.90	2346.3	2618.2	8.8930	6.9383	7.8313	
30	69.10	0.001022	5.22918	289.18	2179.2	2468.4	289.21	2336.1	2625.3	9.9439	6.8247	7.7686	
35	72.69	0.001024	4.52568	304.20	2168.8	2473.0	304.23	2327.1	2631.4	9.9875	6.7282	7.7157	
40	75.87	0.001026	3.99345	317.51	2159.5	2477.0	317.55	2319.2	2636.7	1.0258	6.6441	7.6700	
45	78.73	0.001028	3.57633	329.50	2151.1	2480.6	329.54	2312.0	2641.5	1.0600	6.5697	7.6298	
50	81.33	0.001030	3.24034	340.42	2143.4	2483.8	340.47	2305.4	2645.9	1.0910	6.5029	7.5939	
60	85.94	0.001033	2.73193	359.77	2129.8	2489.6	359.83	2293.6	2653.5	1.1452	6.3867	7.5319	
70	89.95	0.001036	2.36495	376.61	2117.9	2494.5	376.68	2283.3	2660.0	1.1918	6.2878	7.4797	
80	93.50	0.001039	2.08720	391.56	2107.2	2498.8	391.64	2274.1	2665.7	1.2328	6.2017	7.4345	
90	96.71	0.001041	1.86945	405.03	2097.6	2502.6	405.13	2265.7	2670.8	1.2694	6.1254	7.3948	
100	99.62	0.001043	1.69400	417.33	2088.7	2506.1	417.44	2258.0	2675.5	1.3025	6.0568	7.3593	
p (MPa)	100	99.62	0.001043	1.69400	417.33	2088.7	2506.1	417.44	2258.0	2675.5	1.3025	6.0568	7.3593
	125	105.99	0.001048	1.37490	444.16	2069.3	2513.5	444.30	2241.1	2685.3	1.3739	5.9104	7.2843
	150	111.37	0.001053	1.15933	466.92	2052.7	2519.6	467.08	2226.5	2693.5	1.4335	5.7897	7.2232
	175	116.06	0.001057	1.00363	486.78	2038.1	2524.9	486.97	2213.6	2700.5	1.4848	5.6868	7.1717
	200	120.23	0.001061	.88573	504.47	2025.0	2529.5	504.68	2202.0	2706.6	1.5300	5.5970	7.1271
	225	124.00	0.001064	.79325	520.45	2013.1	2533.6	520.69	2191.3	2712.0	1.5705	5.5173	7.0878
	250	127.43	0.001067	.71871	535.08	2002.1	2537.2	535.34	2181.5	2716.9	1.6072	5.4455	7.0526
	275	130.60	0.001070	.65731	548.57	1992.0	2540.5	548.87	2172.4	2721.3	1.6407	5.3801	7.0208
	300	133.55	0.001073	.60582	561.13	1982.4	2543.6	561.45	2163.9	2725.3	1.6717	5.3201	6.9918

Pr. 4

Fill a Rigid Vessel with Air: A rigid vessel of volume 0.2 m^3 initially at vacuum is placed in the environment at *STP* (*= Standard Temperature and Pressure*). At time zero the valve is opened and the tank fills until the final pressure in the tank equals the atmospheric pressure. Start your analysis with the flow forms of mass, energy and second law given on the cover page.

You are to calculate the final temperature in the tank for following two cases.

- (1/2 credit) Adiabatic throughout the fill. Find the irreversibility during the process, δP_2 . The definition of this quantity is given on the formula sheet (see Flow Second Law, Entropy and Real Process).
- (1/2 credit) Reversible throughout the fill. Find the heat exchange during the process, δQ_2 .



Closed book and notes.

Possibly useful information and formulas:

1. **Ideal gas equation of state:** $pV = mRT$ or $pv = RT$ or $p=\rho RT$ where $R = R_u/MW$. We have $R_u = 8.313 \text{ kJ/kmole-K}$. For air $MW = 29 \text{ kg/kmole}$ so $R_{air} = 0.287 \text{ kJ/kg-K}$. If the gas is not ideal we use $pv = ZRT$ and then employ the compressibility charts.
2. **Properties:** For air $c_v = 0.7165 \text{ kJ/kg-K}$, $c_p = 1.0035 \text{ kJ/kg-K}$
3. **Work:** ${}_1W_2 = \int_1^2 pdV$. The work is positive if *by* system *on* surroundings.
4. **Mass Conservation:** $\frac{dm_{CV}}{dt} = \sum_{in} \dot{m}_{in} - \sum_{out} \dot{m}_{out}$
5. **1st Law:** $\Delta E = E_2 - E_1 = {}_1Q_2 - {}_1W_2$ where $E = U + KE + PE$ and $U = mu$. The units of U are energy, u is energy/mass. Note that ${}_1Q_2 = \int_1^2 \delta Q$ and ${}_1W_2 = \int_1^2 \delta W$.
6. **Flow 1st Law:** $\frac{dE_{CV}}{dt} = \dot{Q} - \dot{W} + \sum_{in} \dot{m}_{in}(h + \frac{V^2}{2} + gz)_{in} - \sum_{out} \dot{m}_{out}(h + \frac{V^2}{2} + gz)_{out}$
Note: \dot{Q} is *into* the CV.
7. **Flow 2nd Law:** $\frac{dS_{CV}}{dt} = \dot{P}_{SV} + \sum \frac{\dot{Q}}{T_Q} + \sum_{in} \dot{m}_{in}s_{in} - \sum_{out} \dot{m}_{out}s_{out}$. T_Q is the temperature of the surroundings that exchanges heat with the CV..
8. **Entropy: Definition:** $S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T} \right)_{reversible}$. Also, $dS = \frac{\delta Q}{T}$.
9. **Real Process:** We write $S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T} \right) + {}_1P_2$ where $S = m \cdot s$ is the total entropy (s is entropy per unit mass) with $[s] = \text{kJ/kg-K}$ and $[S] = \text{kJ/K}$. The term ${}_1P_2$ is the entropy generation.
10. **Carnot 2-T Engine:** $Efficiency = \eta = 1 - Q_{cold}/Q_{hot} = 1 - T_{cold}/T_{hot}$.
11. **Gibbs TdS Equation:** $dU = TdS - pdV$. For an incompressible substance $dV = 0$ leaving $dS = dU/T = CdT/T$, which integrates to yield $S_2 - S_1 = C \ln(T_2/T_1)$. For a gas, use of the ideal gas equation of state gives $s_2 - s_1 = c_v \ln(T_2/T_1) + R \ln(v_2/v_1)$. Note that $U = H - pV$ so the TdS equation may also be written as $TdS = dH - Vdp$. When $dS = 0$ this gives $dH = Vdp$ (extensive form) or $dh = vdp$ (intensive form).