

Exam Number: \_\_\_\_\_

Department of Mechanical Engineering  
Michigan State University

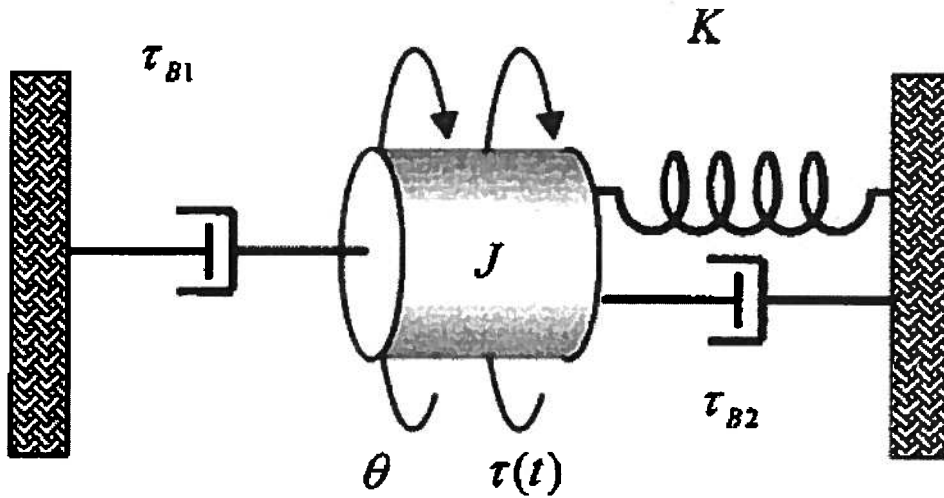
Ph.D. Qualifying Examination  
August 2009  
Dynamic Systems and Control

Open Book  
Answer All Questions  
All Questions Weighted Equally

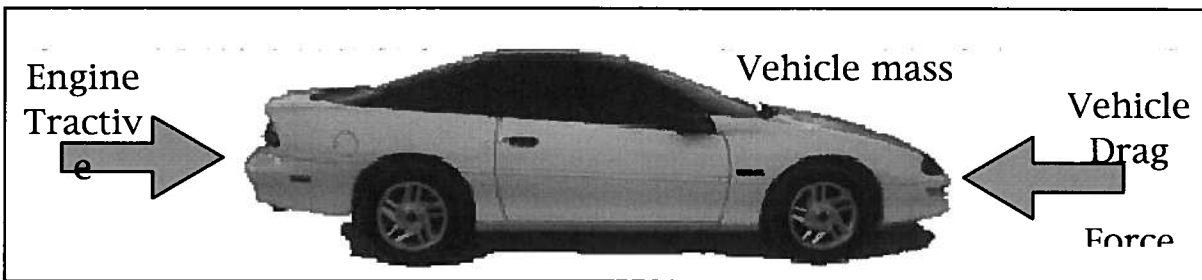
Exam Prepared by

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- 1) Consider the following mechanical system where  $\tau(t)$  is the applied input torque;  $\tau_{B1}(t) = 10\dot{\theta}(t)$  and  $\tau_{B2}(t) = -15\dot{\theta}(t)$  are damping forces; and  $K=20$  and  $J=2$  are the spring stiffness and inertia, respectively.



- Draw the free body diagram of the system.
- Derive the equations of motion
- Determine the transfer function from the input  $\tau(t)$  to the output  $\theta(t)$ , i.e.,  $\frac{\Theta(s)}{T(s)}$
- Determine the system stability from the transfer function.



2) The nonlinear dynamics of a 1995 Camaro are governed by the ordinary differential equation

$$100\dot{v} + 46.87 - 1.558v + 0.02267v^2 = e[229.2 - 0.560v]$$

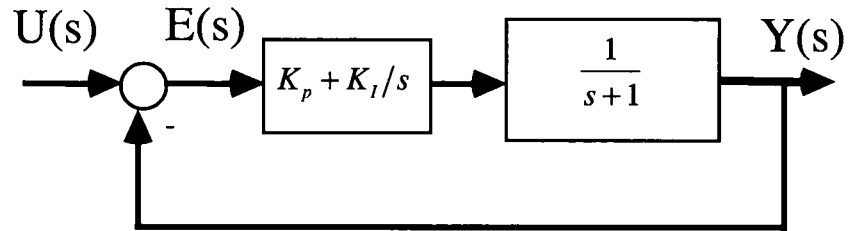
where the vehicle model variables are velocity,  $v = v(t)$  in ft/sec and engine throttle,  $e = e(t)$  in percent.

- a) Determine the linearized differential equation relating  $\dot{v}$  to  $e(t)$  for the equilibrium operating point at  $v_0 = 88 \text{ ft/sec}$ . Hint: the linearized equations are written using deviation variables such as  $\bar{v} = (v - v_0)$  and  $\bar{e} = (e - e_0)$ . (Attach calculations)

- b) Determine the steady state response  $\bar{v}_{ss}$  (ft/sec) of the linearized system found above to the excitation  $\bar{e}_{ss} = 5\%$

$\bar{v}_{ss} =$ 
ft/sec

3) You are to design a Proportional + Integral (PI) controller for the system shown below.



a) Find gains  $K_p$  and  $K_I$  that render the system stable and have a maximum system time constants of  $\tau_{\max} = 0.5$  sec (All others are smaller) and has a closed loop error of less than 1%.

$K_p =$   
  
 $K_I =$

b) At the gains you supplied in part b), what is the steady state error for a unit step input to this system?

4) Sketch bode diagrams for the following transfer functions:

a)  $G(s) = -10$

b)  $G(s) = 1/s^2$

c)  $G(s) = 1/(s+1)$