Exam Number: ____________________

Department of Mechanical Engineering

Michigan State University

Ph.D. Qualifying Examination
January 2004
Dynamic Systems and Control

Open Book, Open Notes

Answer All Questions
All Questions Weighted Equally

Exam Prepared by

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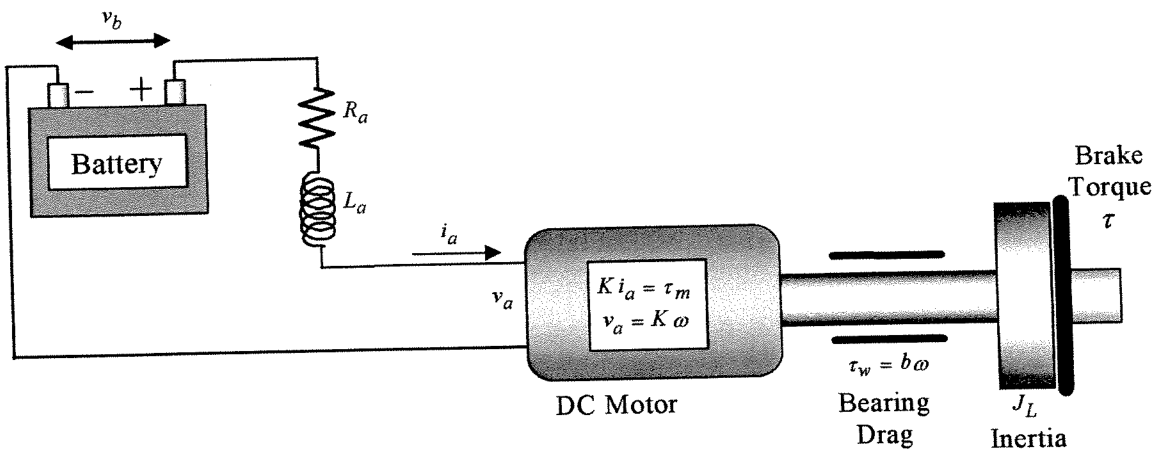
(revised January 5, 2004)

1) Shown below is a schematic diagram for a permanent magnet DC motor with load. The excitation voltage v_b is applied to the motor's armature that has a resistance R_a, and inductance L_a. The motor characteristics include a mechanical torque proportional to armature current $\tau_m = Ki_a$ and an electrical back-emf voltage proportional to motor speed $v_a = K\omega$. The motor torque drives a load that is represented by both viscous friction arising from viscous bearing "drag" and inertia J_L. The motor speed is controlled by the action of a brake with an applied torque τ.



a) Determine the differential state equation(s) that govern the dynamics of this permanent magnet DC motor model. First, identify the inputs and outputs. Then, write the equations in matrix form.

b) Determine the transfer function from the input $\Gamma(s) = \mathcal{L}(\tau)$ to the output $\Omega(s) = \mathcal{L}(\omega)$.

2) The dynamics of a system are governed by the ordinary differential equation

$$\ddot{y}(t) + \dot{y}(t) + \sin(y(t)) = \sqrt{u(t)}$$

which may also be written in the form

$$\ddot{y}(t) = f(u, y, \dot{y})$$

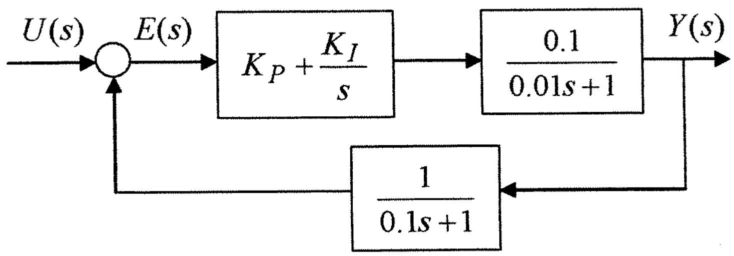
a) Let $v(t) = u(t) - u_0(t)$ and $x(t) = y(t) - y_0(t)$ and determine the linearized differential equation relating $v(t)$ to $x(t)$ for the equilibrium given by $u_0(t) = 1/2$, $y_0(t) = \pi/4$.

b) Determine the steady state response $x_{ss}(t)$ of the linearized system found above to the excitation

$$v(t) = 3 + 5\cos(10t)$$

$x_{ss}(t) =$

3) You are to design a Proportional + Integral (PI) controller for the system below.



a) Determine the range of gains K_I and K_P that will lead to a stable closed-loop system.

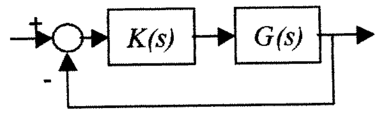
_______ $< K_I <$ _______
_______ $< K_P <$ _______

b) Draw a root locus for this system with $0 < K_P < \infty$ and $K_I = 0$. Use the root locus to find all stable values of K_P for which the system has a 2% settling time less than 1 sec.

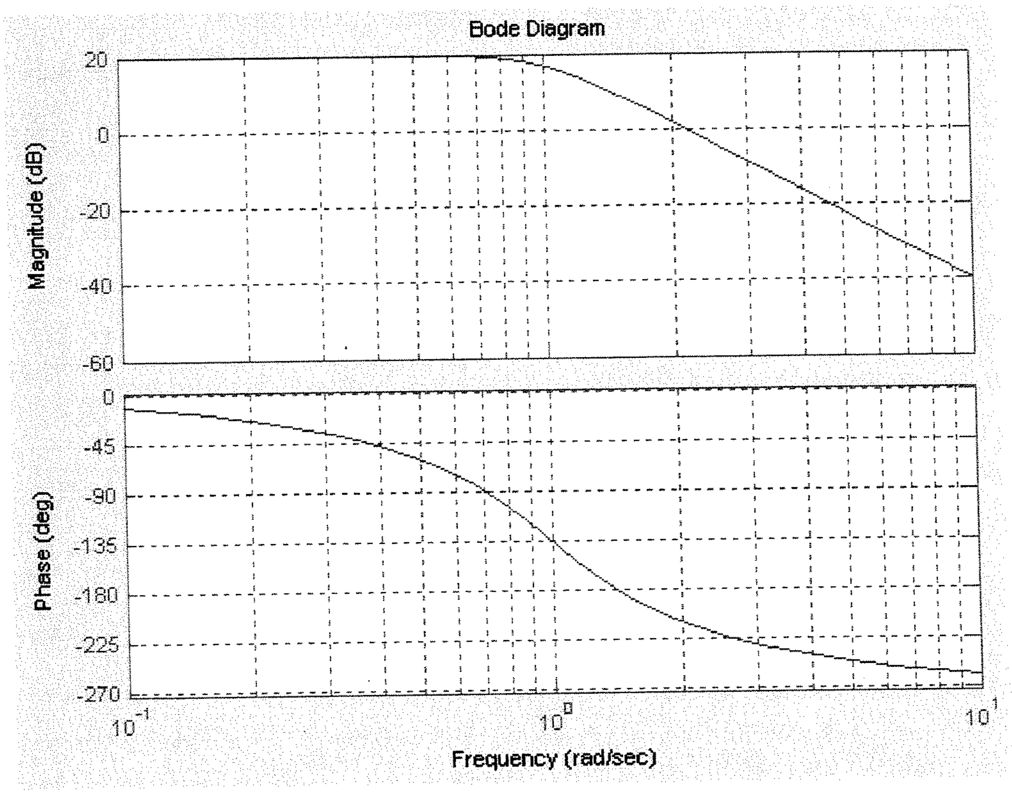
c) For $K_P = 5$, draw a root locus for this system with $0 < K_I < \infty$.

d) With $K_P = 5$ and $K_I = 1$, will this system necessarily respond like a prototype second order system? Why or why not?

4) For the feedback system below,



the Bode diagram $G(j\omega)$ for industrial process that is open-loop stable is drawn below. A Bode diagram is the open-loop frequency response.



a) What are the gain and phase margins of this system? Will it be stable when connected in the above feedback system with $K(s) = 1$?

Gain Margin = Phase Margin =	Stable when K(s) =1? Yes or No (Circle one)

b) Design the "best" (widest bandwidth) Proportional feedback control system that will yield a phase margin of at least 45° and a gain margin of at least 8 dB.

$K(s) = K_P =$

c) Given the design in part (b) above, what is the expected steady state error of the closed-loop system due to a unit step input?

Steady-state error = (%)