Student #_____

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Ph.D. Qualifying Exam in Systems and Controls

- One open book.
- Open Notes.
- Answer all questions.
- All questions carry the same weight.

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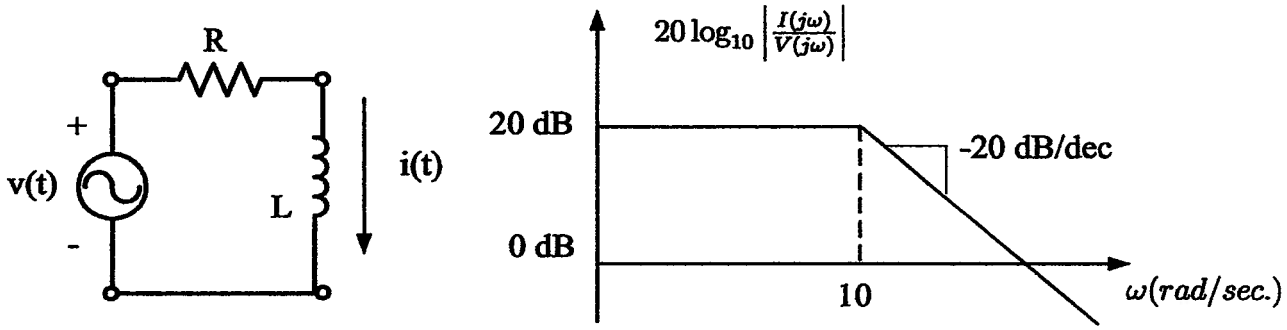
1 Problem



(a) Circuit

(a) Gain Bode Plot of TF I(s)/V(s)

Figure 1: RL Circuit.

2 Problem

Consider a simple pendulum shown in Fig. 2. The mass of the pendulum bob is m, the rod is massless, and assume acceleration due to gravity to be g. u denotes the applied input torque.

a. Derive the equation of motion of the pendulum.

b. Find the nominal input u_0 in order to keep the equilibrium point at the operating point $\theta_0 = \pi/4$.

c. Let $\bar{\theta} := \theta - \theta_0$ and $\bar{u} := u - u_0$. Obtain the linearized equation of motion of the pendulum in terms of $\bar{\theta}$ and $\bar{u}$ about the operating point $\theta_0 = \pi/4$.

d. Let $L = 1/\sqrt{2}, m = 2, g = 10$. Determine the transfer function of the linearized system based on $\bar{u}$ and $\bar{\theta}$, i.e., $G(s) := \frac{\mathcal{L}(\bar{\theta}(t))}{\mathcal{L}(\bar{u}(t))}$. Discuss the stability of the linearized system based on the transfer function $G(s)$.

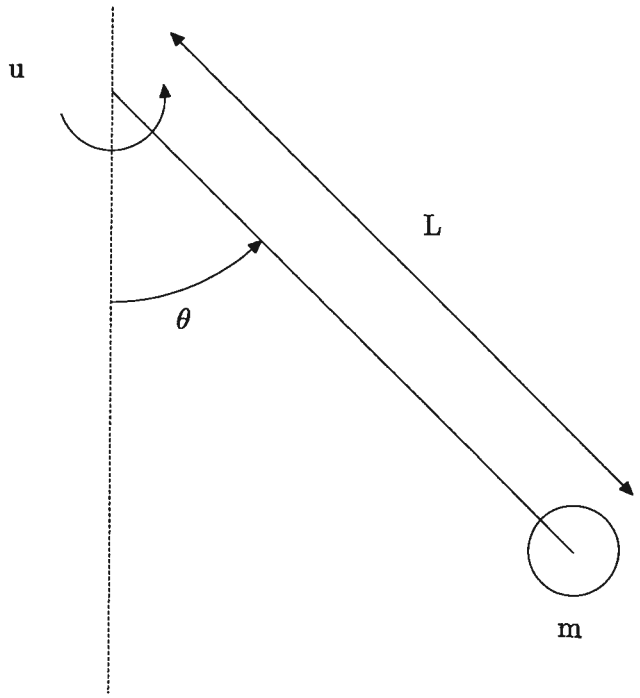


Figure 2: A simple pendulum

4 Problem

A second-order system has the transfer function

$$G(s) = \frac{a_1}{s^2 + b_2 s + b_3}$$

(a) Determine the values of a_1, b_2, and b_3, given the following information:
   (i) The DC gain of the system is 2.0.
   (ii) The undamped natural frequency of the system is 2 rad/sec.
   (iii) The output of the system lags behind a sinusoidal input of frequency 4 rad/sec by 135°.

(b) What will be the percentage overshoot of the system to a step input ?