Exam Number: ______________________________

Department of Mechanical Engineering

Michigan State University

Systems and Control
Ph.D. Qualifying Examination

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Open Book Open Notes
All questions are weighted equally.

Prepared by

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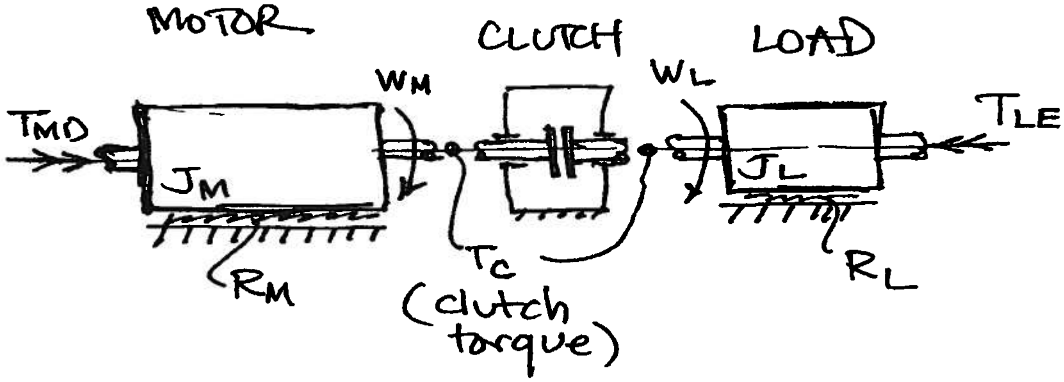
1) Shown below is a schematic diagram of a motor unit driving a load through a clutch device. The driving torque on the motor mechanical subsystem is denoted by $T_{MD}(t)$. The electrical subsystem of the motor is ignored. The external torque on the load is $T_{LE}(t)$.



MOTOR: $\omega_M(t)$, motor angular velocity

Parameters: J_M, motor inertia; R_M, motor friction (viscous)

CLUTCH: $T_C(t)$, torque transmitted by clutch

Open state: $T_C = 0$; Closed state: $T_C = r_C(\omega_M - \omega_L)$

LOAD: $\omega_L(t)$, load angular velocity

Parameters: J_L, load inertia; R_L, load friction (viscous)

(a) Assume the CLUTCH is open. Write a set of differential equations describing the system.

(b) Find the LOAD transfer function $\omega_L(s)/T_{LE}(s)$, assuming the CLUTCH is open.

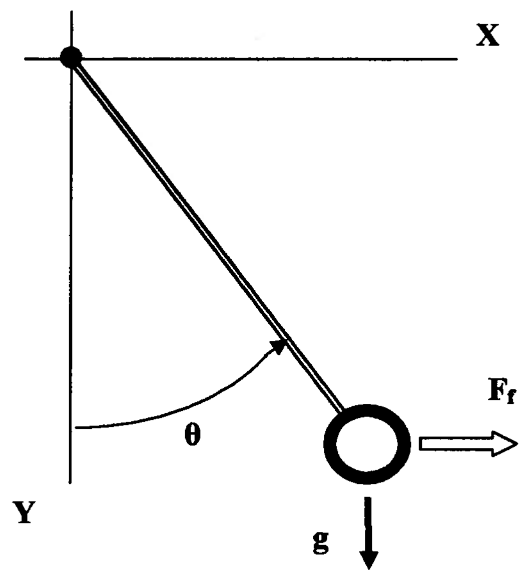
(c) Assume the CLUTCH is closed. Write a set of differential equations describing the system.

(d) Find the system transfer function $\omega_L(s)/T_{MD}(s)$, assuming the CLUTCH is closed. Let the load torque T_{LE} be zero.

(e) Let both T_{MD} and T_{LE} be constant (but not necessarily the same value). Assume the CLUTCH is closed. Find the steady-state value of the LOAD motion, $\omega_L(\infty)$.

(Hint. Check your result for the case that $T_{MD} = T_{LE}$ to show it is reasonable.)

2) A simple pendulum is shown below in a horizontal force field. The constant force is F_f in the $+X$ direction. Gravity 'g' acts in the Y direction. The length of the pendulum is L; the mass is M. The pendulum rotates about the pivot with angular position $\theta(t)$.



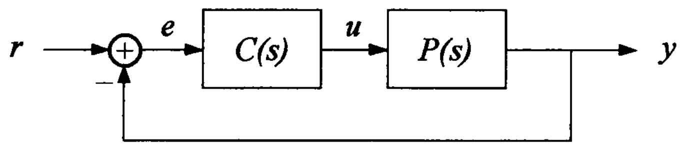
The governing equation is: $ML^2 \dfrac{d^2\theta}{dt^2} = -MgL\sin\theta + F_f L\cos\theta$

(a) If $F_f = Mg$, find the equilibrium point, θ_{EP}. Is it unique?

(b) For small excursions about θ_{EP} from (a), find the linearized system equation. What is your idea of "small excursions"?

(c) What is the natural frequency of the linearized system from (b)? Is it different from the case of an equilibrium point for $F_f = 0$?

3) Consider the control system shown below with $P(s) = \dfrac{(s+4)}{s(s+1)(s+2)}$.



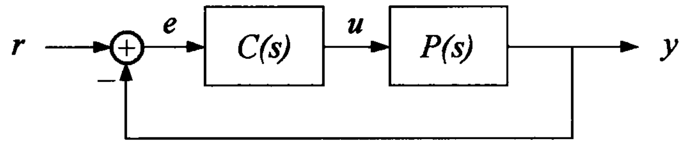
(a) Determine if the plant $P(s)$ is stable.

(b) When $C(s) = K$, sketch the root locus of the system and find the range of K for which the closed-loop system is stable.

(c) When $C(s) = 9$, find the steady-state error due to a unit step input.

(d) Design a PD controller so that the closed-loop system is stable and has a pole at $s = -1 + j$.

4) Consider the control system shown below with $P(s) = \dfrac{1000}{s(s+1)(s+10)}$.



(a) Sketch the Bode plot of $P(s)C(s)$ when $C(s) = 1$.

(b) From the Bode plot, determine approximately the gain crossover frequency, phase crossover frequency, gain margin and phase margin. Is the closed-loop system stable?

(c) Design a controller that satisfies the following design specifications: the steady-state error due to a ramp input is zero, the gain margin is at least 20 dB and the phase margin is at least 45°.