Exam Number: ____________________

Department of Mechanical Engineering

Michigan State University

Ph.D. Qualifying Examination
August 2005
Dynamic Systems and Control

Open Book, Open Notes

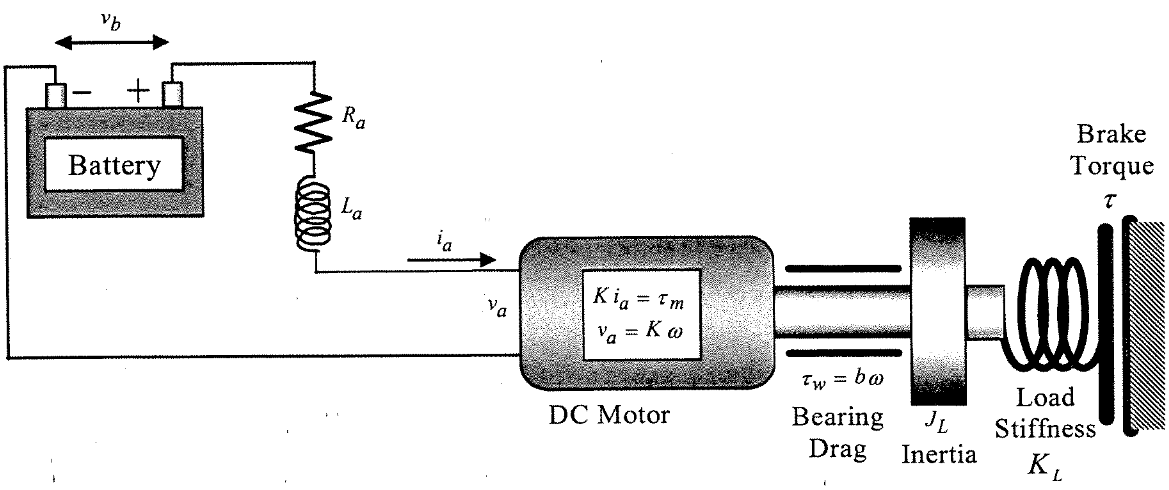
Answer All Questions
All Questions Weighted Equally

Exam Prepared by

Clark J. Radcliffe
Ranjan Mukherjee

(revised May 3, 2005)

1) Shown below is a schematic diagram for a permanent magnet DC motor with load. The excitation voltage v_b is applied to the motor's armature that has a resistance R_a, and inductance L_a. The motor characteristics include a mechanical torque proportional to armature current $\tau_m = Ki_a$ and an electrical back-emf voltage proportional to motor speed $v_a = K\omega$. The motor torque drives a load that is represented by viscous bearing "drag" friction, inertia J_L and load stiffness K_L. The motor speed is controlled by the action of a brake with an applied torque τ through the load stiffness K_L.



a) Determine the differential state equation(s) that govern the dynamics of this permanent magnet DC motor model. First, identify the inputs and outputs. Then, write the equations in matrix form.

b) Determine the transfer function from the input $\Gamma(s) = \mathcal{L}(\tau)$ to the output $\Omega(s) = \mathcal{L}(\omega)$.

2) The dynamics of a system are governed by the ordinary differential equation

$$\ddot{y}(t) + 5(\dot{y}(t))^2 + \sin(y(t)) = \sqrt{u(t)}$$

a) Find the linearized differential equation relating for small deviations in $y(t)$ and $u(t)$ about an operating point where $y_0(t) = \pi/4$.

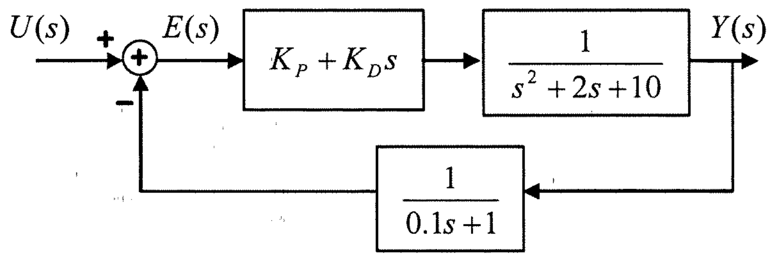
b) For an operating point $(u_0, y_0) = (0.5, 0.707)$, find the steady-state response $\bar{y}_{steady-state} = (y_{steady-state} - y_0)$ of the linearized system found above to the excitation.

$$u(t) = 3 + 5\cos(10t)$$

Note: "Steady-state" is defined as system response after sufficient time has elapsed so that all transients due to initial conditions have died out.

$x_{ss}(t) =$

3) You are to design a Proportional + Derivative (PD) controller for the system below.

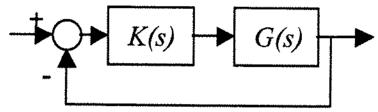


a) Determine the range of gains K_P and K_D that will lead to a stable closed-loop system.

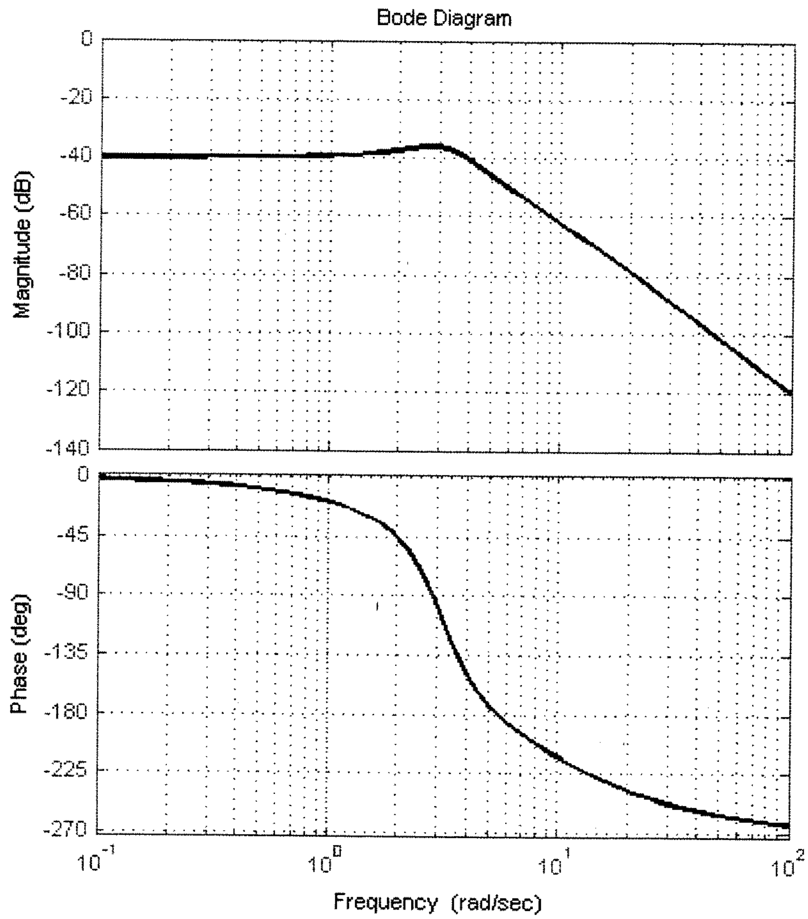
b) Draw a root locus for this system with $0 < K_P < \infty$ and $K_D = 0$. Use the root locus to find <u>all</u> stable values of K_P for which the system has a 2% settling time less than 1 sec.

c) Design PD controller by finding K_P and K_D that yields a closed loop system with a 2 second settling time and 5% overshoot. .

4) For the feedback system below,



the Bode diagram $G(j\omega)$ for industrial process is drawn below.



a) What are the gain and phase margins of this system? Will it be stable when connected in the above feedback system with $K(s) = 1$?

Gain Margin =

Phase Margin =

Stable when K(s) =1? Yes or No
(Circle one)

b) Design the "best" (widest bandwidth) Proportional feedback control system that will yield a phase margin of at least 45° and a gain margin of at least 8 dB.

$K(s) = K_P =$

c) Given the design in part (b) above, what is the expected steady state error of the closed-loop system due to a unit step input?

Steady-state error = (%)