Exam Number: ______________

Department of Mechanical Engineering
Michigan State University

Dynamic Systems and Control Ph.D. Qualifying Examination

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Open Book, Open Notes

Four Questions

All Questions are Weighed Equally

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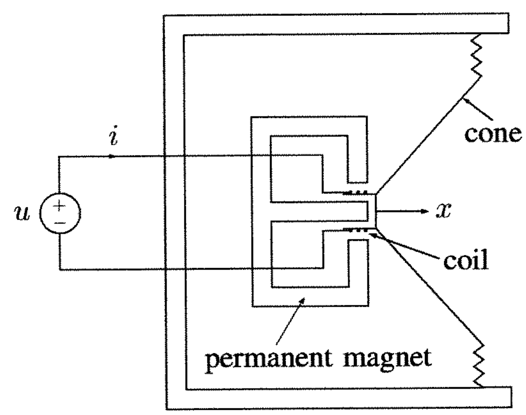
1. A typical geometry for a voice coil motor (loudspeaker) commonly used in hard disks is shown below. When a current i passes through the coil, a magnetic force $F = Ci$ in the x direction is applied on the coil causing it to move, where C is a constant. When the coil moves with a velocity v in the magnetic field, a back emf $e = Cv$ is generated across the coil that opposes the current flow. The effects of the air and conic membrane can be modeled as an equivalent mass M, damper B and spring K. The resistance of the coil is R and the inductance of the coil is L. The voice coil motor is driven by the input u and its output is x.



(a) Write the differential equations that governs the dynamics of the system.

(b) Determine a state-space representation of the system.

(c) Find the transfer function from u to x.

(d) Find x in the steady-state when u is a unit step.

2. Consider the system

$$\ddot{y}(t) + 3y(t)\dot{y}(t) + 4\arctan[y(t) - u(t)/2] = 0, \ t > 0,$$
$$y(0) = 1/2,$$
$$\dot{y}(0) = 1/5,$$

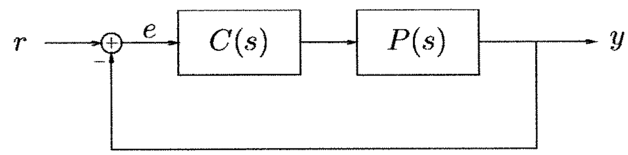
where $u(t)$ is the input and $y(t)$ is the output.

(a) Let $u(t) = U(t) = 2$ for $t > 0$ and determine $y(t) = Y(t)$ for $t > 0$ at equilibrium.

(b) Let $u(t) = U(t) + \delta u(t)$ and $y(t) = Y(t) + \delta y(t)$, where $U(t) = 2$ for $t > 0$ and $Y(t)$ as determined above. Linearize the differential equation about $U(t)$ and $Y(t)$. Note that $d[\arctan(x)]/dx = 1/(1+x^2)$.

(c) Determine $y(t)$ approximately using linearization when $u(t) = 2 + \cos(t)$, $t > 0$.

3. Consider the control system shown below with $P(s) = \dfrac{s+4}{s(s+2)}$.



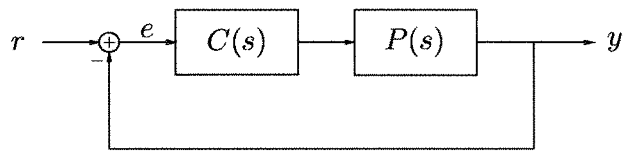
(a) When $C(s) = K_P + K_I/s$, determine all values K_P and K_I for which the closed-loop system is stable.

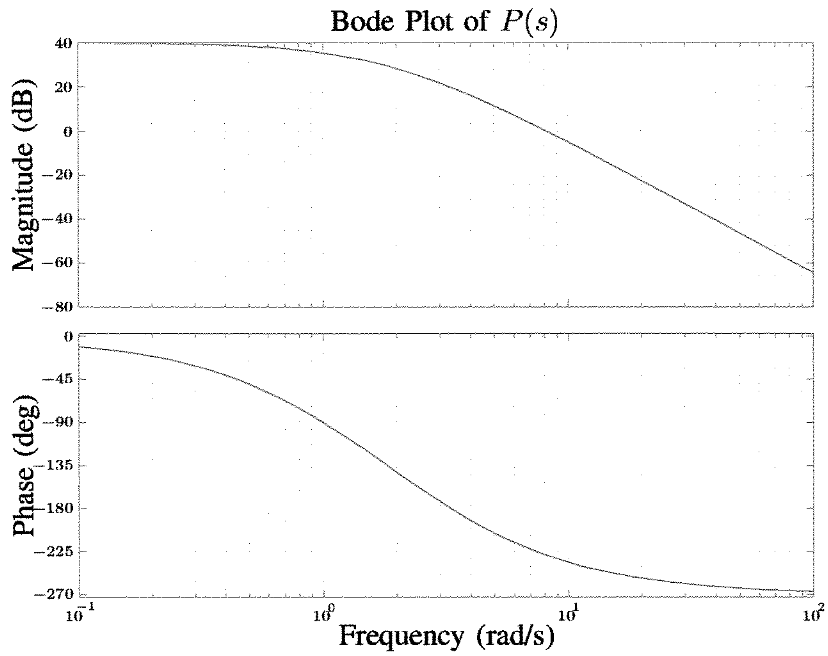
(b) When $C(s) = K_P + K_D s$, determine all values K_P and K_D for which the closed-loop system is stable.

(c) Accurately sketch the root locus of the system when $C(s) = K$.

(d) Design a PI controller so that the closed-loop system has a pole at $s = -1$ and the steady-state error for a unit parabolic input is $1/3$.

4. Consider the control system shown below with an open-loop stable plant $P(s)$ whose Bode plot is as depicted.





(a) Determine the gain margin, phase margin, gain crossover frequency and phase crossover frequency of the system when $C(s) = 1$.

(b) Sketch the Nyquist plot of $P(s)C(s)$ when $C(s) = 1$.

(c) Is the closed-loop system stable when $C(s) = 1$?

(d) Design a proportional controller $C(s) = K$ that minimizes the steady-state error due a unit step and achieves at least 6 dB gain margin and 25 deg phase margin.