

MICHIGAN STATE UNIVERSITY
Department of Mechanical Engineering
Systems and Control Ph.D. Qualifying Examination

Open Book. Open Notes. All problems are weighted equally.
Prepared by Ranjan Mukherjee and Jongeun Choi

Exam Number:

1 Problem

Consider the following transfer function with an unknown gain and corner frequencies:

$$G(s) = k_1 \frac{(s/\omega_1) + 1}{(s/\omega_2) + 1}.$$

The bode plot of $G(s)$ is given in Fig. 1. As you can see in Fig. 1, corner frequencies are either 1 or 10 rad/sec., i.e., $|\omega_i| \in \{1, 10\}$ for $i \in \{1, 2\}$.

- a. Given the conditions and Fig. 1, determine k_1 , ω_1 and ω_2 .
- c. Discuss the stability of $G(s)$.
- d. Consider a negative feedback system with $G(s)$ and a gain k (Fig. 2). Determine the range of the gain value $k \geq 0$ for which the closed-loop system is stable.

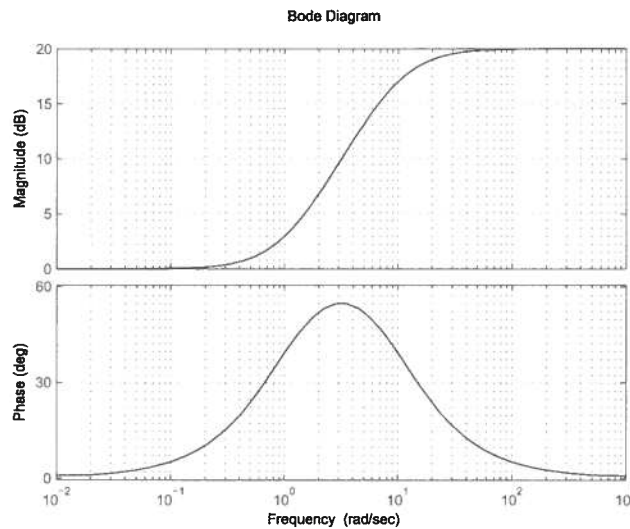


Figure 1: Bode plot of $G(s)$

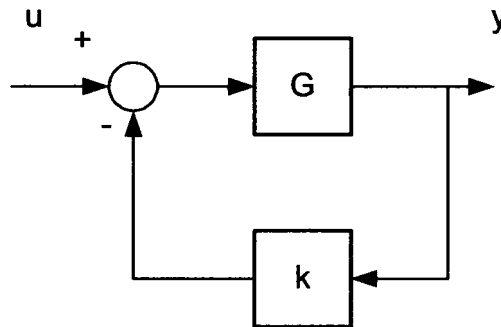


Figure 2: Closed-loop system

2 Problem

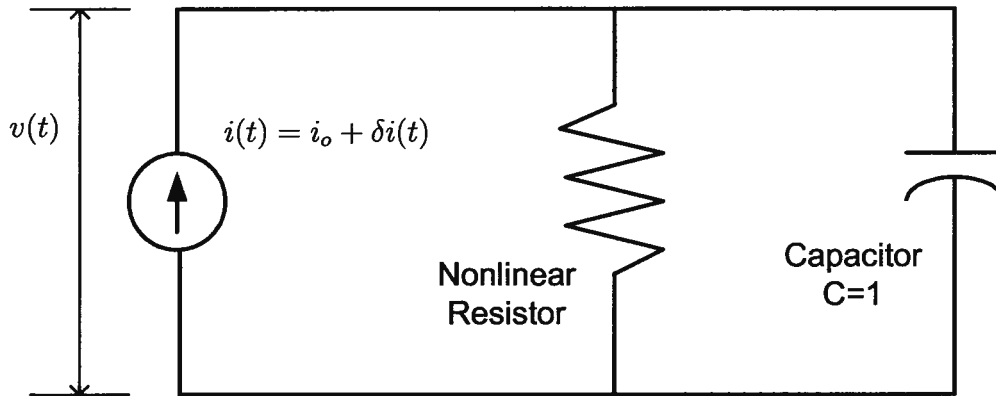


Figure 3: Nonlinear RC Circuit.

Consider the nonlinear electrical network (Fig. 3). The current source to the network is represented by $i(t) = i_o + \delta i(t)$, where $i_o = 2$ denotes the nominal current supply; and $\delta i(t)$ represents the small perturbation. The nonlinear resistor has the voltage-current relationship define by $i_r(t) = e^{v_r(t)}$, where $v_r(t)$ is the voltage drop as the current $i_r(t)$ flows across the nonlinear resistance. The linear capacitor has the capacitance of 1.

Please answer the following questions.

- Derive the nonlinear differential equation of the nonlinear network.
- Find the equilibrium point of $v(t)$ for $\delta i(t) = 0$.
- Let the equilibrium point of $v(t)$ be v_o . Let $\delta v(t) := v(t) - v_o$ and $\delta i(t) := i(t) - i_o$. Obtain the linearized differential equation of the circuit in terms of $\delta v(t)$ and $\delta i(t)$ about the operating point (v_o, i_o) .
- View $\delta i(t)$ and $\delta v(t)$ as the input and the output of the system respectively. Determine the transfer function of the linearized system from $\delta i(t)$ to $\delta v(t)$, i.e., $G(s) := \frac{\mathcal{L}(\delta v(t))}{\mathcal{L}(\delta i(t))}$. Assume that $\delta i(t)$ is small. Discuss the transient responses of the nonlinear electrical circuit for different values for i_o .

3 Problem

Consider the system described by the block diagram below (Fig. 4) with (a) $G(s) = K_p$, (b) $G(s) = sK_d$, and (c) $G(s) = K_i/s$.

Determine the range of values of K_p , K_d , and K_i for cases (a), (b), and (c), respectively, for which the closed-loop system will be stable.

For each of the three cases, namely, (a), (b), and (c), comment on whether the closed-loop system is under-damped, critically-damped, or over-damped when it is stable.

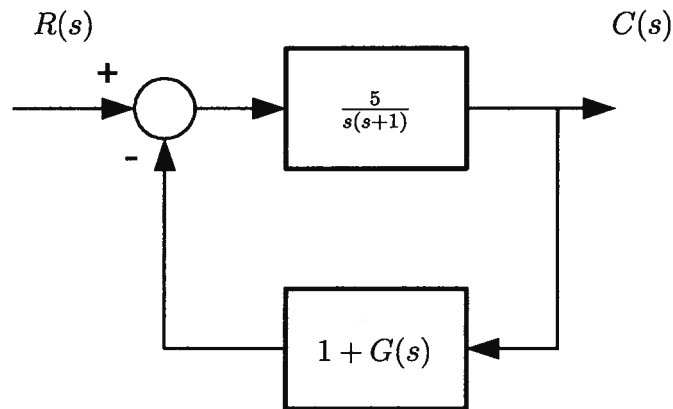


Figure 4: Closed-loop system

4 Problem

For the system described by the transfer function $G(s) = \frac{C(s)}{R(s)} = \frac{5}{s+3}$, find the expression for the output $c(t)$ when the input $r(t)$ has the form $r(t) = 2.0 \sin(2t)$. Determine the steady state response of the system and verify your results from the gain and phase of $G(j2)$.

