Department of Mechanical Engineering
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Systems and Control
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Open Book Open Notes
All questions are weighted equally.

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1) A mechanical schematic for a print-head mechanism is shown in Figure 1-1. The particular variables of interest are the position and motion of the pin, x and v, respectively. Assume that the neutral position is at \( x = 0 \), with the pin centered subject to zero applied force \( F_4 \). The force-deflection characteristics of the platen (at \( x=1 \)) and the hard stop (at \( x=-1 \)) are given by the \( \{ F_2 \text{ vs. } x \} \) relations as if they were a single spring.

![Figure 1-1. Print-head mechanism.]

\[
\begin{align*}
F_2 &= 20.0 \times (x-1), & x > 1.0 \\
F_2 &= 0.0, & -1.0 \leq x < 1.0 \\
F_2 &= 20.0 \times (x+1), & x < -1.0
\end{align*}
\]

Parameters are given as follows (assume they are compatible SI units):

- \( m \), mass of the pin \( \ldots m = 2.0 \)
- \( k_1 \), positioning spring \( \ldots k_1 = 2.0 \)
- \( F_2 \text{ vs. } x \), effect of platen and stop \( \ldots (see \ above) \)
- \( b \), friction effect when pin slides \( \ldots b = 0.2 \)

(a) Is the overall system linear or non-linear? Explain, please.

(b) What constant force \( F_4 \) is required to maintain the pin state as \( \{ x=1, \ v=0 \} \)? Call this value \( F_{crit} \).

(c) Choose \( F_4 = 0.5 \times F_{crit} \). The initial conditions are \( \{ x=0, \ v=0 \} \). Predict the system response as precisely as possible.

(d) Now set \( F_4 = 0.98 \times F_{crit} \). Will your solution from part (b) still apply? Explain.

(e) Now find \( F_4 \) such that the steady-state is \( \{ x=1.5, \ v=0 \} \). The initial conditions are \( \{ x=1.4, \ v=0 \} \). Predict the system response or else explain what difficulties you encounter that prevent that.
2) A long slender shaft rotates in a sleeve, as shown in Figure 2-1. The inputs to the shaft are the driving angular velocity $\omega_D$ and load torque $\tau_L$.

![Figure 2-1. A rotating shaft.](image)

PARAMETERS:
- $J_S = 1.0$ ... inertia of shaft
- $K_S = 9.0$ ... shaft stiffness
- $b_S = 0.1$ ... rotational friction

PART 1.

Assume a so-called “1-lump” model as shown in Figure 2-2.

![Figure 2-2. A “1-lump” model.](image)

(a) If $\omega_D = 10.0$ and $\tau_L = 5.0$, both constant, find the steady-state operating condition of the shaft. If there is a disturbance to the operating condition will there be oscillations in the shaft? Explain, please.

PART 2.

Assume that we wish to use a “2-lump” model (i.e., a model with 2 masses and 2 springs) for improved accuracy. The inputs remain the same as in PART 1.

(b) Sketch a “2-lump” model.

(c) Find the eigenvalues, if you assume that $b_S$ is now zero. How do they relate to the eigenvalues of PART 1?
3) Consider the control system shown below with 

\[ P(s) = \frac{(s + 4)}{(s + \alpha)(s - 2)}. \]

\[ r \rightarrow e \rightarrow C(s) \rightarrow P(s) \rightarrow y \]

(a) When \( C(s) = 1 \), sketch the locus of the closed-loop poles as \( \alpha \) varies in the interval \( 0 \leq \alpha < \infty \) and find the range of \( \alpha \) for which the closed-loop system is stable.

(b) When \( C(s) = 1 \) and \( \alpha = 0 \), find the steady-state error due to a unit step input.

(c) When \( \alpha = 4 \) and \( r(t) = \cos(t)u(t) \), where \( u(t) \) is the unit step function, design a controller so that the output is asymptotically equal to the input. (Hint: Try a controller of the form \( C(s) = (k_1s^2 + k_2s + k_3)/(s^2 + \beta^2) \), where \( k_1, k_2, k_3 \) and \( \beta \) are the design parameters.)
4) Consider the control system shown below with \( P(s) = \frac{100(s + 10)}{(s + 1)(s + 100)} \).

(a) Sketch the Bode plot of \( P(s)C(s) \) when \( C(s) = 1 \).
(b) From the Bode plot, determine approximately the gain crossover frequency, phase crossover frequency, gain margin and phase margin. Is the closed-loop system stable?
(c) Design a controller that satisfies the following design specifications: the steady-state error due to a step input is zero, the gain margin is at least 60 dB and the phase margin is at least 60°.