Exam Number: ______________________________

Department of Mechanical Engineering

Michigan State University

Systems and Control
Ph.D. Qualifying Examination

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Open Book Open Notes
All questions are weighted equally.

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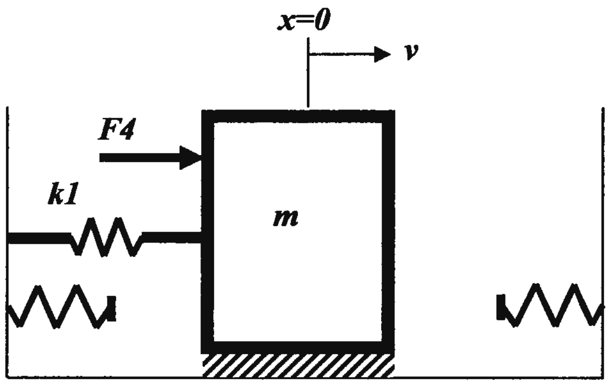
1) A mechanical schematic for a print-head mechanism is shown in Figure 1-1. The particular variables of interest are the position and motion of the pin, x and v, respectively. Assume that the neutral position is at x = 0, with the pin centered subject to zero applied force F4. The force-deflection characteristics of the platen (at x=1) and the hard stop (at x=-1) are given by the {F2 vs. x} relations as if they were a single spring.



F2 = 20.0*(x-1),	x > 1.0
F2 = 0.0,	-1.0 < x < 1.0
F2= 20.0*(x+1),	x < -1.0

Figure 1-1. Print-head mechanism.

Parameters are given as follows (assume they are compatible SI units):

- m, mass of the pin ... *m= 2.0*
- k1, positioning spring ... *k1= 2.0*
- F2 vs. x, effect of platen and stop ... *(see above)*
- b, friction effect when pin slides ... *b= 0.2*

(a) Is the overall system linear or non-linear? Explain, please.

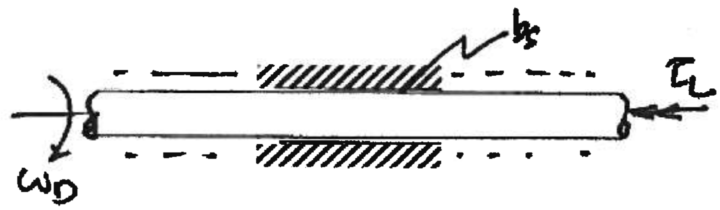
(b) What constant force ***F4*** is required to maintain the pin state as {***x=1, v=0***}? Call this value ***Fcrit***.

(c) Choose ***F4= 0.5*Fcrit***. The initial conditions are {***x=0, v=0***}. Predict the system response as precisely as possible.

(d) Now set ***F4= 0.98*Fcrit***. Will your solution from part (b) still apply? Explain.

(e) Now find ***F4*** such that the steady-state is { ***x=1.5, v=0*** }. The initial conditions are { ***x=1.4, v=0*** }. Predict the system response or else explain what difficulties you encounter that prevent that.

2) A long slender shaft rotates in a sleeve, as shown in Figure 2-1. The inputs to the shaft are the driving angular velocity ω_D and load torque τ_L.



PARAMETERS:
$J_S = 1.0$ … inertia of shaft
$K_S = 9.0$ … shaft stiffness
$b_S = 0.1$ … rotational friction

Figure 2-1. A rotating shaft.

PART 1.

Assume a so-called "1-lump" model as shown in Figure 2-2.

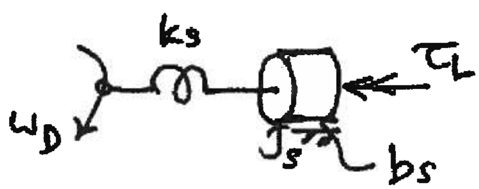


Figure 2-2. A "1-lump" model.

(a) If $\omega_D = 10.0$ and $\tau_L = 5.0$, both constant, find the steady-state operating condition of the shaft. If there is a disturbance to the operating condition will there be oscillations in the shaft? Explain, please.

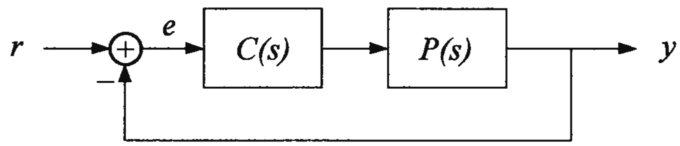
PART 2.

Assume that we wish to use a "2-lump" model (i.e., a model with 2 masses and 2 springs) for improved accuracy. The inputs remain the same as in PART 1.

(b) Sketch a "2-lump" model.

(c) Find the eigenvalues, if you assume that b_S is now zero. How do they relate to the eigenvalues of PART 1?

3) Consider the control system shown below with $P(s) = \dfrac{(s+4)}{(s+\alpha)(s-2)}$.

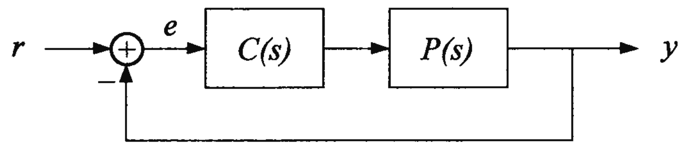


(a) When $C(s) = 1$, sketch the locus of the closed-loop poles as α varies in the interval $0 \le \alpha < \infty$ and find the range of α for which the closed-loop system is stable.

(b) When $C(s) = 1$ and $\alpha = 0$, find the steady-state error due to a unit step input.

(c) When $\alpha = 4$ and $r(t) = \cos(t)u(t)$, where $u(t)$ is the unit step function, design a controller so that the output is asymptotically equal to the input. (Hint: Try a controller of the form $C(s) = (k_1 s^2 + k_2 s + k_3)/(s^2 + \beta^2)$, where k_1, k_2, k_3 and β are the design parameters.)

4) Consider the control system shown below with $P(s) = \dfrac{100(s+10)}{(s+1)(s+100)}$.



(a) Sketch the Bode plot of $P(s)C(s)$ when $C(s) = 1$.

(b) From the Bode plot, determine approximately the gain crossover frequency, phase crossover frequency, gain margin and phase margin. Is the closed-loop system stable?

(c) Design a controller that satisfies the following design specifications: the steady-state error due to a step input is zero, the gain margin is at least 60 dB and the phase margin is at least 60°.