

Student Code Number: _____

Ph.D. Qualifying Exam

Dynamic Systems and Control

Fall 2019

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Professor Michael Lavagnino

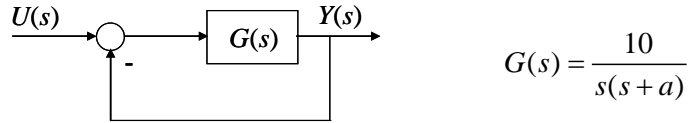
Department of Mechanical Engineering
Michigan State University

Open Book (one book allowed)
Answer All Questions
All Questions Weight Equally

Network Devices Are Not Allowed
All Work Should Be Shown For Full Credit

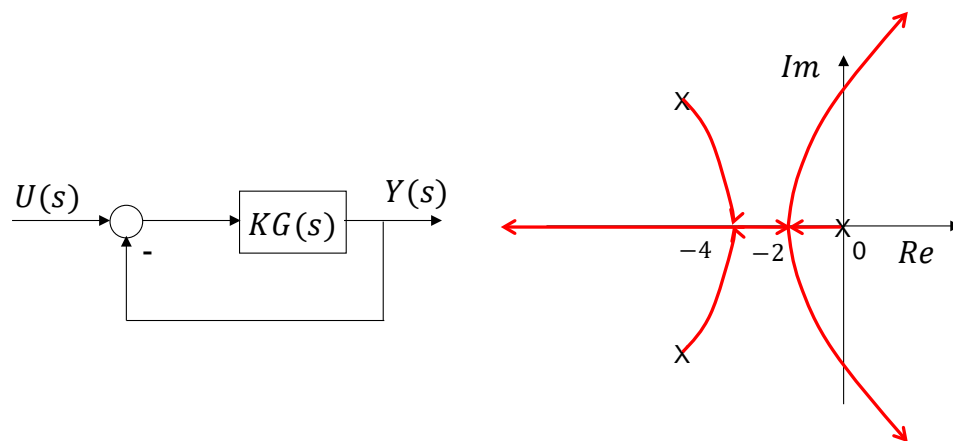
Time: 3.0 hours

Consider the following unit negative feedback system



- Find the closed-loop transfer function $G_{CL}(s) = \frac{Y(s)}{U(s)}$.
- Calculate ramp-error constant K_v as a function of $a > 0$ and find “ a ” such that the steady-state error (e_{ss}) with respect to unit ramp input is 0.5.
- Using “ a ” found in b), calculate the corresponding closed-loop system damping coefficient (ζ) and natural frequency (ω_n).
- Find the closed-loop system 2% settling time $t_s(2\%)$ and percentage of overshoot (σ).

2. For the closed-loop system shown below with the corresponding Root Locus, when $K = 10$ the closed-system is marginally stable. Find the unique transfer function $G(s)$.



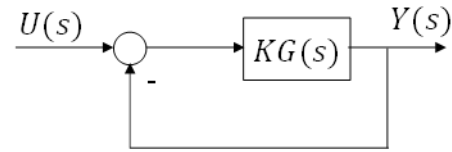
3. A researcher determines the following non-linear differential equation model of soft tissue strain ϵ versus stress values, σ (MPa) during loading.

$$\dot{\epsilon} = -8\epsilon - \epsilon^3 + 4\epsilon\sigma$$

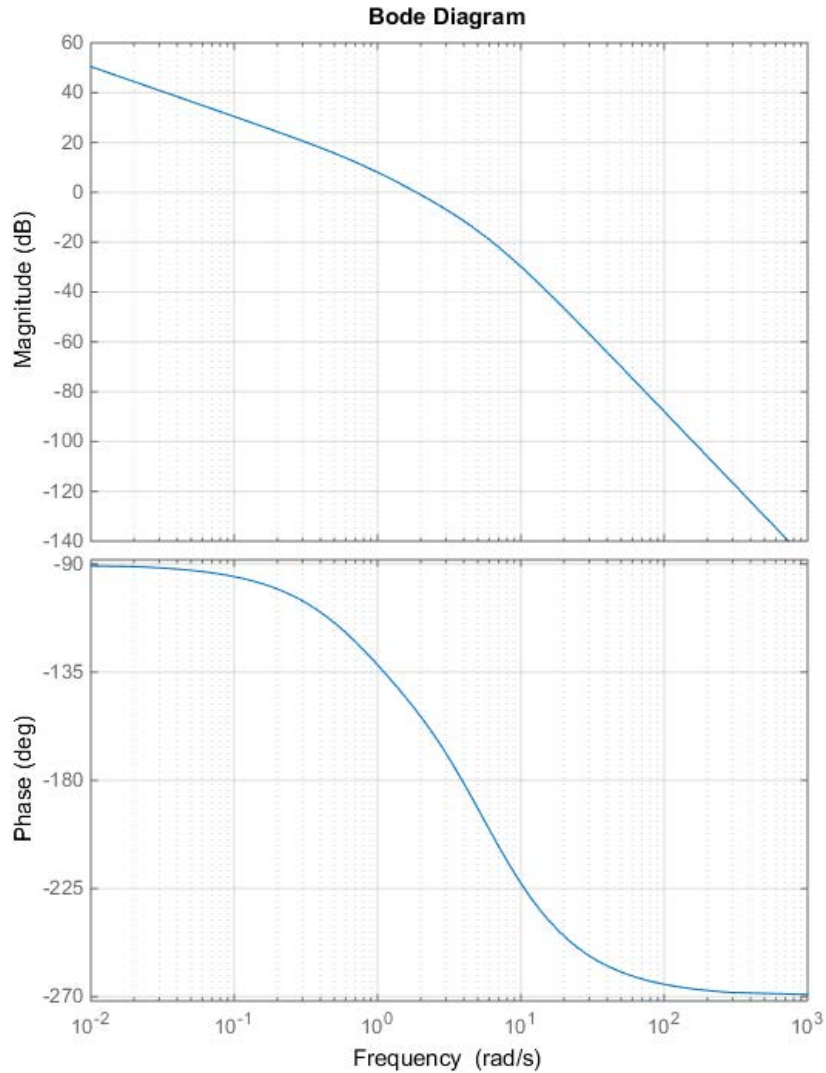
- a) Find the equilibrium stress value σ_0 corresponding to a steady strain value of $\epsilon_0 = 2$.

- b) Find the linearized ordinary differential equation about the equilibrium point, found in a), in terms of $\delta\epsilon = \epsilon - \epsilon_0$ and $\delta\sigma = \sigma - \sigma_0$.

4. Consider the negative unit feedback system with a transfer function $G(s)$ as in the figure where $K=40$.



The bode plot of $KG(s)$ is given as follows:



- Estimate the gain margin (dB) and the phase margin (deg) directly from the bode plot. Justify your answers, i.e., explain how they are obtained from the bode plot.
- Given that $K=40$ in the bode plot above, find the maximum integral value of K for which the closed-loop transfer function will remain stable.