Student Code Number:_____

Ph.D. Qualifying Exam

Dynamic Systems and Control

Spring 2020

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Open Book (one book allowed)
Answer All Questions
All Questions Weight Equally

Time: 3.0 hours

1. Consider a mechanical system that satisfies the following differential equation

$$\ddot{x} + 5\dot{x} + 10x + 4x^5 + \frac{2}{\pi} sin\left(\frac{x\pi}{2}\right) = u$$

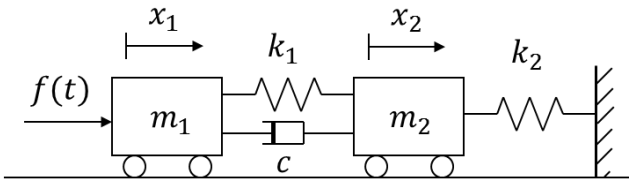
a) Linearize the above nonlinear system at $x_0 = 1, \ddot{x}_0 = \dot{x}_0 = 0$ with $x = x_0 + \delta x$ and $u = u_0 + \delta u$ such that the resulting differential equation is a function of δx and δu.

b) Find the corresponding linearized transfer function $\frac{\Delta X(s)}{\Delta U(s)} = \frac{L[\delta x(t)]}{L[\delta u(t)]}$

c) Calculate the linearized system damping ratio ς and natural frequency ω_n

Sol.

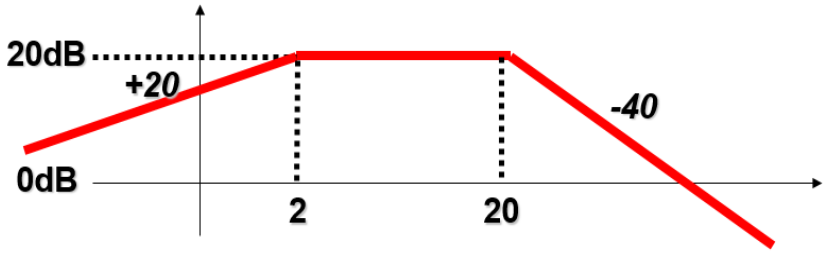
2. Consider two carts moving on a table (see below), where the table surface is assumed to be frictionless.



a) Draw free body diagrams for both M_1 and M_2
b) Find the differential equations for both free body diagrams using Newton's law, and then find the Laplace transforms of both differential equations
c) Find transfer functions from $F(s)$ to $X_1(s)$ and from $F(s)$ to $X_2(s)$.

Sol:

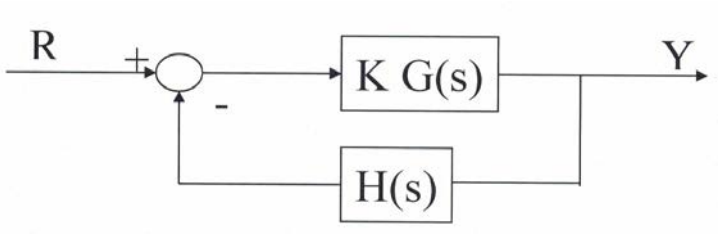
3. Consider the following Bode gain plot.



a) Let the system be stable. Find its transfer function.
b) Let the system be unstable. Find a transfer function. Is this transfer function unique?

Sol:

4. Sketch the root locus for the following system



where $G(s)=\frac{(s+1)}{s^2}$, $H(s)=\frac{1}{(s+9)}$

Sol: