Exam Number: -----------------------------

Department of Mechanical Engineering

Michigan State University

Solid and Structural Mechanics
Ph.D. Qualifying Examination

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Closed Book and Notes
All Questions are weighted equally.

Prepared by

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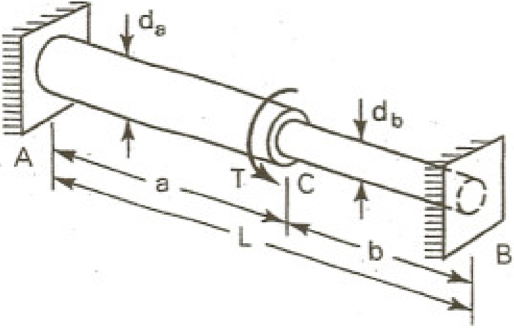
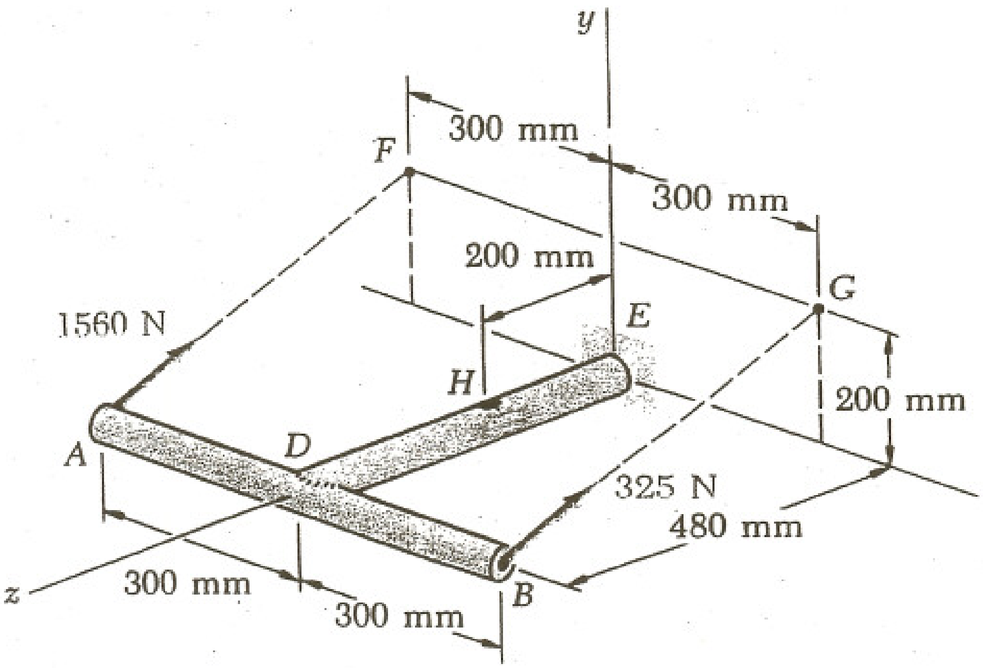
1. A solid circular shaft *AB* is fixed to rigid walls at both ends and subjected to a torque *T* at section *C* as shown in the figure. The shaft diameters are d_a and d_b for segment *AC* and *CB*, respectively. Determine the lengths *a* and *b* if the maximum shearing stress in both the shaft segments is to be the same, for $d_a = 20$ mm, $d_b = 12$ mm and $L = 600$ mm.



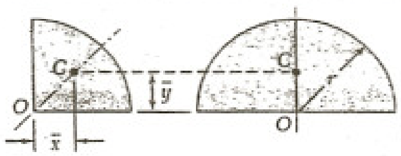
2. Two forces are applied as shown to rod AB, which is welded to the 50-mm-diameter cylinder DE. Assuming that all stresses remain below the proportional limit, determine the principal stresses and the maximum shearing stress at point H on the top surface of the rod AB.

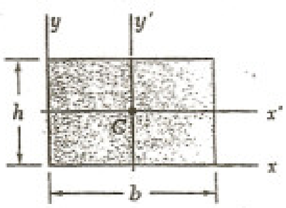
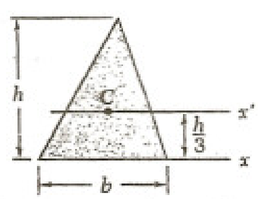
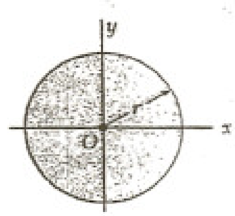


Formula Sheet

Stress : $\sigma = -\frac{My}{\bar{I}}$; $\sigma = \frac{P}{A}$; $\tau = \frac{VQ}{It}$; $\tau = \frac{T\rho}{J}$

Deformation : $\delta = \sum_i \frac{P_i L_i}{A_i E_i}$; $\phi = \sum_i \frac{T_i L_i}{J_i G_i}$; $\frac{1}{\rho} = \frac{d^2 y}{dx^2} = \frac{M(x)}{EI}$

Shape		$\bar{x}$	$\bar{y}$	Area
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$

Rectangle		$\bar{I}_{x'} = \frac{1}{12}bh^3$ $\bar{I}_{y'} = \frac{1}{12}b^3h$ $I_x = \frac{1}{3}bh^3$ $I_y = \frac{1}{3}b^3h$ $J_C = \frac{1}{12}bh(b^2 + h^2)$
Triangle		$\bar{I}_{x'} = \frac{1}{36}bh^3$ $I_x = \frac{1}{12}bh^3$
Circle		$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$ $J_O = \frac{1}{2}\pi r^4$

3. Find the reaction forces in the following beam fixed at both ends A and B by using the following formulae: (1) equilibrium equations, (2) compatibility equations and (3) the principle of superposition. **Hint:** *Based on the superposition of three simply supported beams whose deflection formulae need to be identified first.*