Ph.D. Qualifying Exam
SOLIDS & STRUCTURAL MECHANICS

Spring 2014

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Directions: Closed book and notes, you may use a one page (8.5x11) – one sided formula sheet.

Answer all four questions are weighted equally

Time: 3.0 hrs.
Determine the reaction forces in sections, AB, AC and AD and displacements at A, C and D of the bar shown below. Please indicate the direction of displacements (up or down). The section AB is made of a tube with outer and inner diameters of 6cm and 3cm, respectively. The section B has been mounted on rigid wall as shown. In the diagram, the sections AC and AD are made of a 2cm diameter solid bar and connected by the rigid bar (shown at A). The elastic modulus is 20GPa.
A cylindrical pressure vessel whose radius and thickness are 20cm and 2cm, respectively, is carrying 2MPa of pressure. The vessel is made with the material that has $E=30\text{ GPa}$ and $\nu=0.3$. A strain gage is mounted at 45° to monitor the pressure. Determine the reading on the strain gage.
For full credit, all work must be shown. Free Body Diagrams must also be included for full credit.

3. Several forces are applied to the pipe assembly shown. Knowing that the inner and outer diameters of the pipe are equal to 1.50 inches and 1.75 inches respectively, determine a) the forces and moments acting at point H located at the top of the outside surface of the pipe, b) the principal planes and the principal stresses at H, b) the maximum shearing stress at the same point.
4. For the beam and loading shown, determine the equation of the elastic curve, the deflection at B and the deflection at C.
Determine the reaction forces in sections, AB, AC and AD and displacements at A, C and D of the bar shown below. Please indicate the direction of displacements (up or down). The section AB is made of a tube with outer and inner diameters of 6cm and 3cm, respectively. The section B has been mounted on rigid wall as shown. the sections AC and AD are made of a 2cm diameter solid bar and connected by the rigid bar(shown at A). The elastic modulus is 20GPa.

\[ R_{AB} = 50N \]
\[ R_{AC} = 150N \]
\[ R_{AD} = 100N \]

\[
\sigma_{AB} = \frac{-50}{\pi(0.03^2 - 0.015^2)}
\]
\[
\sigma_{AC} = \frac{150}{\pi(0.01^2)} = 477.5kPa
\]
\[
\sigma_{AD} = \frac{100}{\pi(0.01^2)} = 318.3kPa
\]
\[ \sigma_0 = \frac{Pr}{t} = \frac{2(20)}{2} = 20 \text{MPa} \]
\[ \sigma_z = \frac{Pr}{2t} = \frac{2(20)}{4} = 10 \text{MPa} \]

\[ \varepsilon_r = \frac{\sigma_r}{E} - \nu \frac{\sigma_{\theta}}{E} - \nu \frac{\sigma_z}{E} = -0.3 \left( \frac{20}{30} + \frac{10}{30} \right) = -300 \mu \]
\[ \varepsilon_{\theta} = \frac{\sigma_{\theta}}{E} - \nu \frac{\sigma_r}{E} - \nu \frac{\sigma_z}{E} = \frac{20}{30} - 0.3 \frac{10}{30} = 566.7 \mu \]
\[ \varepsilon_z = \frac{\sigma_z}{E} - \nu \frac{\sigma_{\theta}}{E} - \nu \frac{\sigma_r}{E} = \frac{10}{30} - 0.3 \frac{20}{30} = 133.3 \mu \]

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**Reading on Strain gage = 350 \mu**
\[ r_0 = c_0 = \frac{1.75}{2} = 0.875 \]
\[ r_i = c_i = \frac{1.5}{2} = 0.750 \]
\[ F_x = 50 \text{ lb} \]
\[ M_x = 30(8) + 30(8) = 480 \]
\[ M_y = 50(10) = 500 \]
\[ J = \frac{\pi}{3} (0.875^4 - 0.75^4) = 0.4237 \text{ in}^4 \]

**Bending**

- **Top View**

\[ M_y \]

\[ \delta_x = -\frac{M_x y}{I} = -\frac{480 (0.875)}{0.2118} \]

\[ I = \frac{1}{12} J = 0.2118 \text{ in}^4 \]

\[ y = 0.875 \]

\[ \delta_x = -1.982 \text{ lb in} \]

**Torsion**

\[ T = \frac{T_c}{J} \]

\[ T_{xy} = -400 (0.875) \frac{0.4237}{\text{in}^3} = -825.9 \text{ lb in} \]
Shear: \( T_{xz} = \frac{YQ}{I} = \frac{50(0.1654)}{0.218 (280)} \)

\( t = 2(0.875 - 0.75) = 0.250 \)

\( Q = \frac{y}{g} \cdot \text{Area} \)

\( y = \frac{4r_1}{8\pi} - \frac{4r_2}{8\pi} \)

\( \text{AREA} = \frac{1}{2} \pi r_2^2 - \frac{1}{2} \pi r_1^2 \)

\( Q = \frac{1}{3} (r_0^3 - r_i^3) = 0.1654 \)

\( \delta_y = -198.2 \frac{\text{in}}{\text{in}^2} \)

\( \tan 2\theta_p = \frac{2T}{\delta_z - \delta_y} = \frac{2(674.9)}{0 + 198.2} \)

\( \tan 2\theta_p = -0.681 \quad \theta_p = -17.1^\circ \quad \text{or} \quad 163^\circ \)

\( \delta_{\text{ave}} = \frac{1}{2} (\delta_z + \delta_x) = \frac{1}{2} (0 - 198.2) = -99.1 \)

\( R = \sqrt{(\delta_z - \delta_x)^2 + T^2} = \sqrt{991^2 + (674.9)^2} = 1198.1 \)

\( \delta_1 = \delta_{\text{ave}} + R = 207.9 \)

\( \delta_2 = \delta_{\text{ave}} - R = -218.7 \quad \delta_2 = \delta_{\text{ave}} - R = -218.7 \quad T_{\text{max}} = R = 1198 \)
For the beam and loading shown, determine:

a) equation of the elastic curve
b) the deflection at B
c) the deflection at C.

\[ B.C. \]
\[ x = 0, \quad \frac{\partial y}{\partial x} = 0 \]
\[ x = 0, \quad y = 0 \]

\[ \Sigma F_x = 0 \quad \Sigma F_y = 0 \]
\[ R_x = 0 \quad R_y - w(\frac{L}{4}) = 0 \quad R_y = \frac{3wL}{2} \]

\[ q + \Sigma M_a = 0 \]
\[ M_a = w\left(\frac{L}{3}\right)\left(\frac{3L}{4}\right) = 0 \quad M_a = \frac{3}{8}wL^2 \]

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**Section Beam** \( 0 < x < \frac{L}{2} \)

\[ R_y = \frac{wL}{2} \]

\[ M_{max} = 0 \]

\[ M_1 = \frac{wL}{2} (x) + \frac{3}{8}wL^2 \]

\[ M_1 = \frac{wLx}{2} - \frac{3}{8}wL^2 \]
\[ M = \frac{3}{8} \omega L^2 \]

\[ R_y = \frac{\omega L}{2} \]

\[ \frac{1}{2} L < x < L \]

\[ 2M_x = 0 \]

\[ \frac{3}{8} \omega L^2 - R_y(x) + \frac{\omega}{2} (x - \frac{L}{2})^2 + M_2 = 0 \]

\[ \frac{3}{8} \omega L^2 - R_y(x) + \frac{1}{2} \omega (x - \frac{L}{2})^2 + M_2 = 0 \]

\[ M_2 = -\frac{3}{8} \omega L^2 + \frac{\omega L}{2} x - \frac{1}{2} \omega (x - \frac{L}{2})^2 \]

use singularity functions

\[ EI \frac{d^2 y}{dx^2} = M = -\frac{3}{8} \omega L^2 + \frac{\omega L}{2} x - \frac{1}{2} \omega (x - \frac{L}{2})^2 \]

\[ EI \frac{dy}{dx} = -\frac{3}{8} \omega L^2 x + \frac{\omega L}{2} x^2 - \frac{1}{6} \omega (x - \frac{L}{2})^3 + C_1 \]

B.C. \( x = 0 \): \[ \frac{dy}{dx} = 0 \quad x = \frac{L}{2} \]

\[ 0 = 0 + 0 + 0 + C_1 \quad C_1 = 0 \]
\[
\begin{align*}
EIy &= \frac{-3}{16} \frac{WL^2x^2}{E} + \frac{WLx^3}{12} - \frac{1}{24} w \left(x - \frac{L}{2}\right)^4 + C \\
\text{B.C.} \quad x = 0, \ y = 0 \\
0 &= 0 + 0 + 0 + c_2 \quad c_2 = 0 \\
a) \text{ Elastic curve} \\
y = \frac{w}{EI} \left(- \frac{3}{16} \frac{L^2x^2}{E} + \frac{Lx^3}{12} - \frac{1}{24} \left(x - \frac{L}{2}\right)^4 \right) \\
0 = \frac{dy}{dx} = \frac{w}{EI} \left(- \frac{3}{8} \frac{L^2x}{E} + \frac{Lx^2}{4} - \frac{1}{6} \left(x - \frac{L}{2}\right)^3 \right) \\
b) \text{ Deflection @ B. - find } y @ x = \frac{L}{2} \\
y_B = \frac{wL^4}{EI} \left(- \frac{3}{16} \left(\frac{L}{4}\right)^2 + \frac{L}{16} \left(\frac{L}{4}\right) \right) = \frac{wL^4}{EI} \left(-\frac{7}{192}\right) \\
- \frac{3}{64} + \frac{1}{96} = - \frac{3}{64} + \frac{1}{96} = \\
c) \text{ Deflection @ C = L} \\
y = \frac{wL^4}{EI} \left(- \frac{3}{16} \left(\frac{L}{2}\right)^2 + \frac{L}{12} - \frac{1}{24} \left(L - \frac{L}{2}\right)^4 \right) \\
\frac{wL^4}{EI} \left(- \frac{3}{16} + \frac{1}{12} - \frac{1}{24} \left(\frac{L}{16}\right) \right) = - \frac{41}{384} \frac{wL^4}{EI}
\end{align*}
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