Student Code Number:_____

Ph.D. Qualifying Exam

Dynamic Systems and Control

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Open Book
Answer All Questions
All Questions Weight Equally

Time: 3.0 hours

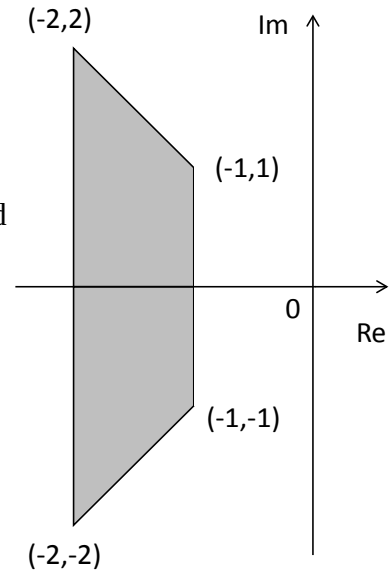
1. A second order system has the following transfer function

$$G(s) = \frac{2\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

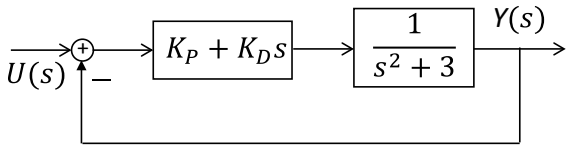
where both poles $s_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$ are located anywhere inside the grey area (see the left figure)

a) Determine the following:
   i. The upper and lower bounds of the percentage overshoot of the system
   ii. The upper and lower bounds of the 2% settling time

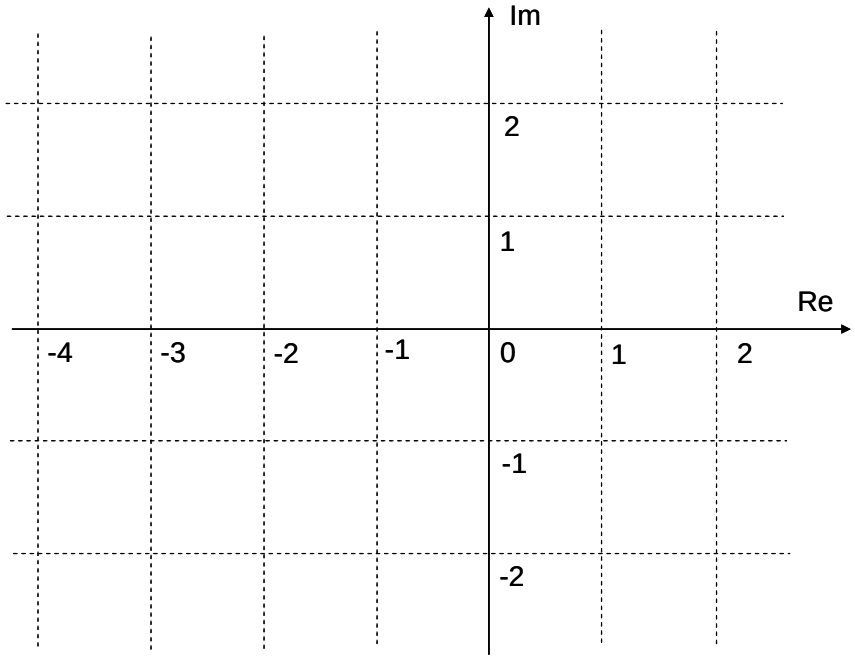
b) Find the all poles inside the grey area with percentage overshoot equal to 2% and draw these poles on the left figure inside the grey area.



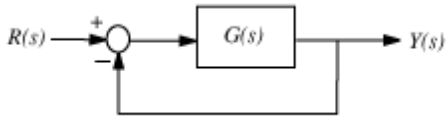
2. Consider the closed-loop system shown in the figure below with a PD controller



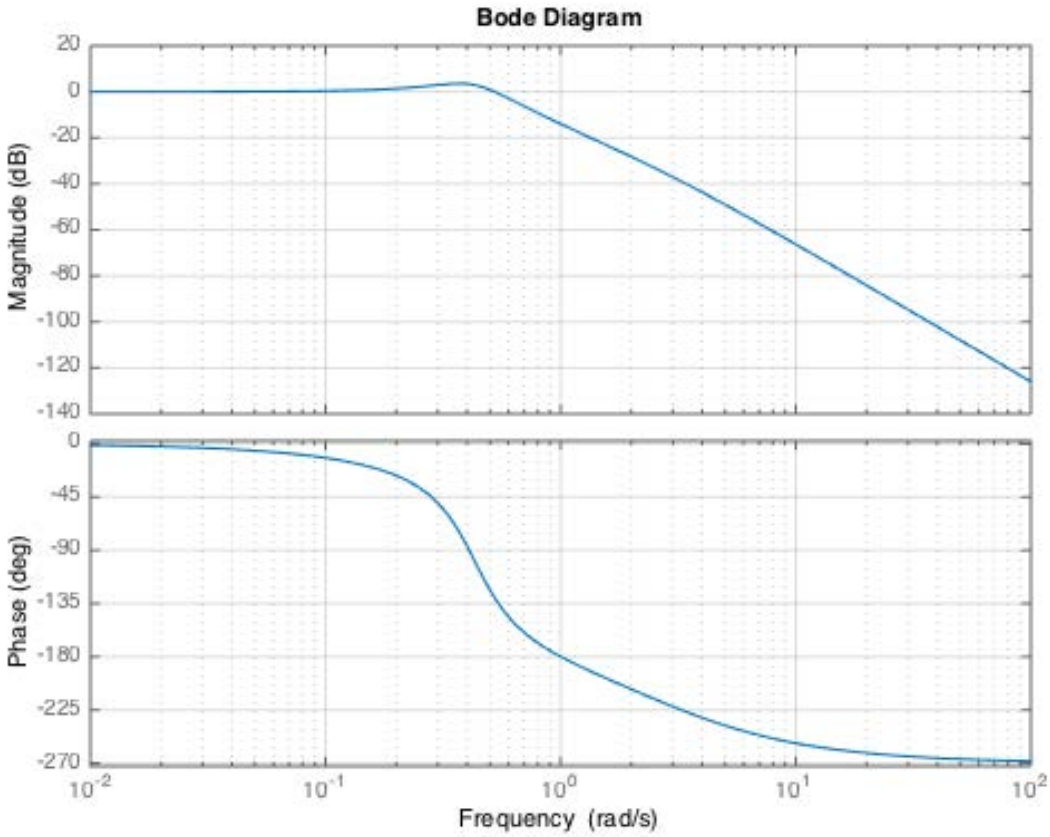
a) Sketch a Root Locus of the closed loop system for $K_P = 1$ with K_D varying from 0 to $2\sqrt{2}$.
b) Sketch another Root Locus of the closed loop system for $K_D = 2\sqrt{2}$ with K_P varying from 1 to ∞. Note that this should form a complete Root Locus.


Im
2
1
Re
-4
-3
-2
-1
0
1
2
-1
-2

3. Consider the negative unit feedback system with a transfer function G(s) in the below figure.



The bode plot of G(s) is given as follows.



(a) Find the gain margin (dB) and the phase margin (deg) directly from the bode plot. Justify your answers, i.e., explain how they are obtained from the bode plot.

(b) Replace G(s) with 0.707G(s). What will happen to gain and phase margins? Justify your answers.

4. Consider the following nonlinear dynamical system.

$$\ddot{x} = x - x^3 - \dot{x} + u \quad ,$$

where x is the state and u is the control variables.

(a) Linearize the nonlinear system at the operating point $x_0 = -1$ with respect to the newly defined coordinates $\delta\ddot{x}, \delta\dot{x}, \delta x, \delta u$.

(b) Find the transfer function from the input δu to the output δx.

(c) Determine the stability of the linearized system. Justify your answer.