Student Code Number: ________________

Ph.D. Qualifying Exam

Mathematics

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Directions: Work all six problems. Problems are equally weighted. One book and one 3 ring binder of notes allowed. No calculators.

Problem 1

Consider the following system of algebraic equations:

$$2x + 3y + 4z = 11$$
$$3x + y + 8z = 0$$
$$x + 7y + 6z = 4$$

a. Write this system in a matrix formulation. (25%)
b. Show that this system has a unique solution. (25%)
c. Obtain the solution to this system. (25%)
d. Determine the eigenvalues for the system. (25%)

Problem 2

Given the following vector function

$$f = [z\cdot\cos(ax)\cdot\sin(by)]\hat{e}_x + [dx^2y^3]\hat{e}_y + [e^{gz}\cdot\sin(ax)]\hat{e}_z$$

where a, b, d, and g are real constant. Determine

a. the divergence of the function (50%)
b. the curl of the function (50%)

Problem 3

Consider a mechanical system modeled by the following differential equation

$$\frac{d^2x}{dt^2} - 5\frac{dx}{dt} + 2x = 8U(t - 6)$$

where U(t-6) is the unit step function (Heaviside unit function) with the step occurring at t=6. Determine the function x(t) given the initial conditions:

$$x(t = 0) = 1 \quad \text{and} \quad \left.\frac{dx}{dt}\right|_{t=0} = 0$$

Problem 4

Find the general or, where possible, the specific solution to each of the following ordinary differential equations. Solutions may be given implicitly or explicitly.

a. $y' + y \cdot \tan(x) = \sec(x)$ (25%)
b. $y' = e^{(x+y)}$ (25%)
c. $y'' + 6y' + 5y = 0$, $y(0) = 3$, $y'(0) = -2$ (25%)
d. $y'' - 8y' + 7y = 4t - \sin t$ (25%)

Problem 5

Find the first three non-zero terms of Taylor series expansions of the following functions about the point $z = a$.

a. $\cos(z^2)$ (40%)

b. $(\cos(z))^2$ (40%)

c. Can you use one result to check the other? If so, explain how. (20%)

Problem 6

Find particular solutions to the partial differential equation for u(x; y):

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 2(x + y)u$$

with u(1; 1) = 1 and u(1; 0) = 1.