Math Qualifying Exam
Department of Mechanical Engineering

January, 2004

Open Book and Open Notes

All questions weighted equally.

JAN '04

1. Write down (but do not solve!) the equations needed to find the function $f(x)$ spanned by the basis $\{1, x, \sin(2\pi x)\}$ that is the best (least square) approximation to the data (x_i, y_i), $i=1,\ldots,N$ in the interval $(0,1)$.

2. Find the point $\mathbf{x}^* = (a, b, c)$ where the function

$$f(x_1, x_2, x_3) = \frac{1}{2}\mathbf{x}^T \begin{bmatrix} 2 & 0 & -1 \\ 0 & 4 & 0 \\ -1 & 0 & 4 \end{bmatrix} \mathbf{x} + \mathbf{x}^T . \{1, -1, 0\}$$

has a minimum along a line that passes through $\{1, 1, -1\}$ along the direction $\{-4,-3,5\}$

3. Find the solution $T(t)$ to the equation

$$aT + b = \frac{dT}{dt}, \quad t>0 \qquad T(0)=T_0$$

where a and b and T_0 are known constants.

4. Find the solution $y(x)$ to the equation

$$\frac{d^4y}{dx^4} = q(x) \quad \text{on } (0,2) \quad \text{where } q(x) = \begin{cases} 1 & \text{if } 0 \le x \le 1 \\ 0 & \text{if } 1 < x \le 2 \end{cases}$$

5. Let $\mathbf{x}=\{ x_1, x_2, x_3\}$ and let

$$K = \begin{bmatrix} 7 & -1 & -4 \\ -1 & 7 & 4 \\ -4 & 4 & 4 \end{bmatrix}$$

Find a (3x3) matrix T such that for any $\mathbf{y} = T^T \mathbf{x}$,

$$\mathbf{y}^T A\, \mathbf{y} = \mathbf{x}^T K\, \mathbf{x}$$

and A is a diagonal matrix. What is A?

6. Using a three-term Taylor series expansion about the point $x = 9$, estimate the value of $f(x) = \sqrt{x}$ at $x = 13$.

7. Find the directional derivative of $f(x) = x + 2y - z$ at the point $(1,4,0)$ in the direction of the vector $\vec{a} = (0,1,-1)$.