Math Qualifying Exam
Department of Mechanical Engineering

January 2009

Open book (only one book allowed)

Answer all questions
All questions have equal weight

TIME: 3.0 hrs

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(In order to receive full credit you must show all work)

Problem #1. Find the polar moment of inertia (J) for an ellipse with semiaxes a and b, where

$$J = \iint \left(x^2 + y^2 \right) dA .$$

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Problem #2. solve the following initial value problem:

$$\frac{dy}{dt} = ay\left(1 - \frac{y}{b}\right), \qquad y(0) = y_o,$$

where a and b are constant.

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Problem #3. The error function is given by

$$\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt .$$

Expand the error function in a Taylor series centered at z=0 up to three non-zero terms.

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Problem #4. Find all the eigenvalues of the following matrix. Then find an eigenvector of length one corresponding to the largest eignevalue.

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 4 & 1 \\ 0 & 3 & 6 \end{vmatrix}$$

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Problem #5. Solve the following initial value problem:

$\frac{dx}{dt}=-4x-y$ and $\frac{dy}{dt}=-6x-3y$ with $x(1)=1, y(1)=0.$

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Problem #6. Consider the function $\mathbf{f}(x,y,z)=3y^2z-2x^2y-6x\cos z$. (a) Find its directional derivative in the direction of $\underset{\sim}{u}=6\hat{\mathbf{i}}-3\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ at the point $(1,1,0)$. (b) Find $\mathbf{div}(\nabla\mathbf{f})$ at the point $(2,1,0)$.

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