Code Number :..............

MATHEMATICS QUALIFYING EXAM

January 2008

OPEN BOOK (only one book allowed)

Answer all questions

All questions have equal weight

TIME: 3.0 hrs

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Problem #1

Consider

$f(x, y, z) = \sin(x^2 + y^2 + z^2) + xy + xz + yz$.

One of its critical point (min, max, or saddle point) is (0,0,0). Identify by computing the variou first- and second-order partial derivatives the nature of the critical point you have identified.

Problem #2

Stokes' Theorem can be stated as

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_S \nabla \times \mathbf{F} \cdot \mathbf{n}\, d\sigma$$

Verify Stokes' theorem using the vector field $\mathbf{F} = (x^2 - y)\mathbf{i} + 4z\mathbf{j} + x^2\mathbf{k}$ where the closed contour is comprised of the x and y coordinate axis and the portion of the circle $x^2 + y^2 = a^2$ that lies in the first quadrant with z=1.

Problem #3

Consider

$$\mathbf{A} = \begin{pmatrix} 2 & -2 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

and find all the eigenvalues of matrix $\mathbf{A}$. Find the eigenspace corresponding to the rea
eigenvalue of $\mathbf{A}$.

Problem 4.

a) What is the general solution of the following ODE?

$$y' - 3y = e^{3x} \sin x$$

b) What is the solution to the following initial value problem, and on what interval is i
defined?

$$y' = x \sin x^2 \,, \;\; y(0) = 3$$

c) Solve the following initial value problem and determine the interval on which th
solution is defined.

$$y' = \frac{y + x}{y - x} \,, \;\; y(2) = 0$$

d) What is the general solution of the following ODE?

$$y' = 2\left(\frac{y}{x}\right)^3 + \frac{y}{x}$$

e) What is the solution of the following ODE?

$$y'' + 4y = 2e^{x} - \sin x$$

Problem 5.

The failure rate of a certain mechanical device has been studied and quantified. The problen of finding the rate dr/dt at which devices should be replaced, to keep a specified numbe $f(t)$ of devices operational at time t, is described by the equation

$$f(t) = f(0)p(t) + \int_0^t p(t-\tau)\frac{dr}{d\tau}\,d\tau \qquad (1$$

where $f(0)$ is the number of new devices installed at time $t = 0$ and $p(t)$ is a function tha determines the device's longevity and takes the form $p(t) = e^{-ct}$.

Using Laplace transformation or otherwise, find the required replacement rate $dr/dt(t$ of devices so that the number of operational devices $f(t)$ is always a constant number A Verify your answer by evaluating (1) in the physical domain.

Explain briefly (but do not attempt to solve) how your solution strategy for this probler might change if $p(t) = c/t$.

Problem 6.

The deviation of fluid pressure p from its ambient value during one-dimensional acoust vibrations is described by the equation

$$c^2 \frac{\partial^2 p}{\partial x^2} - \frac{\partial^2 p}{\partial t^2} = 0$$

where c is the (constant) sound speed in the fluid. The lateral velocity u driven by acoust motions is related to the pressure field by the equation

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x}$$

where ρ is the (constant) fluid density. We consider the well-known case of acoustic vibr tions in a long tube or bottle that is closed at $x = 0$ (where $u = 0$ and $\partial p/\partial x = 0$) an open at $x = L$ (where $p = 0$). Find expressions for the pressure oscillations, and then th velocity oscillations, that arise from these acoustic vibrations.

Suppose these acoustic vibrations are to be damped by placing a fibrous material, tha will retard the fluid motion, at a location at which the fluid velocity is greatest. What the amplitude of the pressure field (the sound you are trying to damp) at this location?