Math Qualifying Exam
Department of Mechanical Engineering

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Closed books and notes.

All questions weighted equally.

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Question 1

Consider the function $f(x,y,z) = x^2 - 2xy + e^{2z}$. Find its directional derivative in the direction $2\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$ a the point $(x,y,z) = (2,-1,0)$. Find $\text{div}(\nabla f)$ at the point $(x,y,z)=(1,1,0)$.

Question 2

Solve the following equation by undetermined coefficients:

$$y'''' - y' = 5 - 4\cos x + 2e^{x}.$$

Question 3

Consider the Fourier cosine series for the function $f(x) = x$ for $0 < x < \pi$. By finding the general formula for a_n (including a_0) find the forth nonzero term in the Fourier cosine series.

Question 4

Find all the eigenvalues of the following matrix. Then find an eigenvector of length one corresponding to the largest eigenvalue.

$$\begin{matrix} 2 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 4 & 1 \end{matrix}$$

Question 5

Solve the following initial value problem:

$dx/dt = -5x - y$ and $dy/dt = 4x - y$ with $x(1) = 0, y(1) = 1$.

Question 6

Use Laplace transform to solve the following integrodifferential equation:

$$y'(t) = 1 - sin(t) - \int_0^t y(\tau)d\tau, y(0) = 0$$