Code No.____________________________

Mathematics Ph.D Qualifier Exam

Department of Mechanical Engineering
Michigan State University

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Directions: One open book permitted

All problems carry equal weight.

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Ph.D. Qualifying Equation
Mathematics

Problem 1

Consider the following system of differential equations:

$$x_1' = x_1 - 3x_2$$
$$x_2' = -2x_1 + 3x_2$$

with

$$x(0) = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

a. Write this system in a matrix formulation. (25%)
b. Obtain the solution to this system. (75%)

Problem 2

Given the following function in cylindrical coordinates

$$f(r,z,\theta) = az^2 + e^{-br}\cos(d\theta)$$

where a, b, and d are real constants. Determine

a. the gradient of the function (50%)
b. the Laplacian of the function (50%)

Problem 3

Use Laplace transforms to solve the following differential equation

$$\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + 5x = 125t^2$$

given the initial conditions:

$$x(t=0) = 0 \quad \text{and} \quad \left.\frac{dx}{dt}\right|_{x=0} = 0$$

Problem 4.

a) What is the general solution of the following ODE?

$$y' + y\cos t = \sin t \cos t$$

b) What is the solution to the following initial value problem, and on what interval is it defined?

$$y' + 2xy = e^{-x^2}\ ,\ y(0) = -1$$

c) Solve the following initial value problem and determine the interval on which the solution is defined.

$$y' = \frac{y+x}{y-x}\ ,\ y(2) = 2$$

d) What is the general solution of the following ODE?

$$y' = \frac{3y^2 - x^2}{2x - y}$$

e) What is the solution of the following ODE?

$$y'' - 2y' - 3y = 3e^{2t}$$

Problem 5.

Find the first three terms of the Taylor series of the given functions about $z = a$.

a)

$$f(z) = \frac{1}{1-z}\ , \quad a = 3i$$

b)

$$f(z) = \tan z\ , \quad a = \frac{\pi}{4}$$

c)

$$f(z) = \frac{2-3z}{2z^2-3z+1}\ , \quad a = -1$$

Problem 6.

Solve the following equation using separation of variables for $u(x, y)$.

$$\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial y^2} = 0, \; 0 < x < 1, \; 0 < y < \frac{1}{2}$$

with $u(0, y) = 1$, $u(1, y) = 1$, $\partial u/\partial y = 0$ at $y = 0$, and $u(x, \frac{1}{2}) = 0$.