MATHEMATICS QUALIFYING EXAM
DEPARTMENT OF MECHANICAL ENGINEERING

AUGUST 2004

Open Book and Open Notes

All questions weighted equally.

Aug, '04

1. Fit an equation of the form $y = b_0 + b_1 x + b_2 x^2$ to the data (i.e., find b_0, b_1, and b_2) using the least squares method.

x	y
1.2	7300
1.9	7500
2.5	7550
3.1	7510
4.0	7200

2. Solve the initial value problem:

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 4\cos x + 3\sin x \quad , \quad y(0) = 1 \quad , \quad \left.\frac{dy}{dx}\right|_{x=0} = 0$$

3. Solve the differential equation:

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

in the domain $0 < x < 10$

for boundary conditions: $u(0,t) = u(10,t) = 0$

and initial conditions $u(x,0) = x(100 - x^2)$

4. Find all the eigenvalues and one eigenvector of the following matrix. Show all work.

$$A = \begin{bmatrix} 26 & -2 & 2 \\ 2 & 21 & 4 \\ 4 & 2 & 28 \end{bmatrix}$$

5. Compute a second-order Taylor expansion of the function $f(x_1, x_2) = x_1^5 x_2^4 - x_2$ about the point (1,2).