Math Qualifying Exam
Department of Mechanical Engineering

August 2009

Open book (only one book allowed)

Answer all questions
All questions have equal weight

TIME: 3.0 hrs

Prepared by: Profs. F. Pourboghrat and S. Baek

(In order to receive full credit you must show all work)

Problem #1. Solve the initial-value problem (for $x>0$)

$$\frac{dy}{dx} - \frac{2}{x}y = x^3 e^x, \qquad y(1)=0$$

Problem #2. Find all the eigenvalues and eigenfunctions for the following boundary value problem.

$$y'' + \lambda y = 0, \qquad y(0) - y'(0) = 0, \qquad y(\pi) - y'(\pi) = 0$$

Problem #3. For the given matrix **A**, find an orthogonal matrix **Q** and a diagonal matrix **D** such that $\mathbf{A} = \mathbf{QDQ}^T$ where det(**Q**)=1 (Hint: One of eigenvalues of **A** is 5.)

$$\mathbf{A} = \begin{bmatrix} 1 & -2 & 2 \\ -2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

Problem #4. Solve the initial value problem:

$$\left(x^2+1\right)y'+y^2+1=0, \qquad y(0)=1$$

Problem #5. (a) Find the directional derivative $\partial f/\partial s$ of $f(x,y,z)=2x^2+3y^2+z^2$ at the point $P:(2,1,3)$ in the direction of the vector $\underset{\sim}{a}=\hat{i}-2\hat{k}$, and (b) Find $\mathbf{div}(\nabla \mathbf{f})$.

Problem #6. The differential equations governing a mechanical system consisting of two masses and two springs are given as:

$$\begin{cases} \ddot{y}_1 = -5y_1 + 2y_2 \\ \ddot{y}_2 = 2y_1 - 2y_2 \end{cases}$$

Solve for $y_1(t)$ *and* $y_2(t)$, the displacements of the masses in terms of time t.