Exam Number: ________________________

Department of Mechanical Engineering

Michigan State University

Mathematics
Ph.D. Qualifying Examination

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One Book, Closed Notes
All six questions are weighted equally
Question 7 is for extra credit

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Question 1

Solve the initial value problem

$$\frac{dx_1}{dt} + 2x_1 - x_2 = 1 + e^{-t}$$

$$\frac{dx_2}{dt} + x_1 + 2x_2 = 3$$

with

$$x_1(0) = \frac{5}{2} \text{ and } x_2(0) = -\frac{1}{2}$$

Questions 2

Determine whether the vectors
$\mathbf{v}_1 = (1,1,0)$, $\mathbf{v}_2 = (1,-2,2)$, and $\mathbf{v}_3 = (3,0,3)$ are linearly independent.

If they are not linearly independent, give the relation among the three vectors and suggest a new vector or vectors which, along with $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$, span R^3.

If they are linearly independent, express the vector (1,1,1) as a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$.

Question 3

Derive the Taylor series of $1/(x-1)$ about $x=4$ and determine the radius of convergence R of the Taylor series expansion.

Question 4

What is the best linear approximation $g(x)$ to the function $f(x) = 2x^3 + 4x^2 - 2$ in the interval [-1,1]? Use a least square approach, i.e., define g to minimize the error $\int_{-1}^{1} (f - g)^2 \, dx$

Question 5

Solve the diffusion equation

$$\frac{\partial u}{\partial t} = k^2 \frac{\partial^2 u}{\partial x^2} \quad \text{for } 0 \le x \le L, \quad t > 0$$

with the boundary conditions

$$u(0,t) = u(L,t) = 0, \qquad t>0$$

and the initial condition

$$u(x,0) = \begin{cases} x, & 0 \le x \le L/2 \\ L-x & L/2 \le x \le L \end{cases}$$

Questions 6

Diagonalize the matrix

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

i.e., find matrices **Q** and **D** such that $\mathbf{Q}^{-1}\mathbf{A}\mathbf{Q}=\mathbf{D}$ and **D** is diagonal. If **A** cannot be diagonalized, explain why.