

Student Code Number: _____

Ph.D. Qualifying Exam

MATHEMATICS

SPRING 2012

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Directions: Only One Book Allowed.

Answer all four questions

All questions have equal weight

Time: 3.0 hrs.

1. Linear Algebra: Show all work – no calculator solutions

a) Find the inverse of the matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 5 \\ 5 & 4 & 0 \end{bmatrix}$$

b) Find the solution x to the set of equations

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 5 \\ 5 & 4 & 0 \end{bmatrix} x = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

c) Find the eigenvalues and eigenvectors of the matrix

$$\begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$$

Solution(s):

2. Ordinary Differential Equations:

- a) Use the Laplace Transform to find the solution to the ODE

$$\frac{dy}{dt} + 4y = 3, \quad y(0) = 1$$

- b) Find the solution $y(t)$ for the system ordinary differential equations

$$\begin{aligned} \frac{dy}{dt} + 4y + x &= 3 \\ \frac{dx}{dt} + x &= 0 \end{aligned} \quad \text{where at } t = 0, \begin{bmatrix} y(0) \\ x(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Note: A Laplace transform table is attached to this exam

Solution(s):

3. Infinite Series: Show all work – no calculator solutions

a) Approximate $f(x)=\sin(x)$ at $a=\pi/6$ using the first four terms of the Taylor Series

b) Find the Fourier coefficients and Fourier series of the function defined by

$$f(x) = \begin{cases} 0 & \text{if } -\pi \leq x < 0 \\ 1 & \text{if } 0 \leq x < \pi \end{cases} \quad \text{and} \quad f(x+2\pi) = f(x)$$

Solution(s):

4. Partial Differential Equations: Show all work – no calculator solutions

Solve $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$ with $u(0,t)=0$, $u(L,t)=0$, $u(x,0)=f(x)$

Solution(s):

Laplace Transform Table:

Time function $e(t)$	Laplace transform $E(s)$
$u(t)$	$\frac{1}{s}$
t	$\frac{1}{s^2}$
$\frac{t^2}{2}$	$\frac{1}{s^3}$
t^{k-1}	$\frac{(k-1)!}{s^k}$
e^{-at}	$\frac{1}{s+a}$
te^{-at}	$\frac{1}{(s+a)^2}$
$t^k e^{-at}$	$\frac{(k-1)!}{(s+a)^k}$
$1 - e^{-at}$	$\frac{a}{s(s+a)}$
$t - \frac{1 - e^{-at}}{a}$	$\frac{a}{s^2(s+a)}$
$1 - (1+at)e^{-at}$	$\frac{a^2}{s(s+a)^2}$
$e^{-at} - e^{-bt}$	$\frac{b-a}{(s+a)(s+b)}$
$\sin bt$	$\frac{b}{s^2+b^2}$
$\cos bt$	$\frac{s}{s^2+b^2}$
$t \sin bt$	$\frac{2bs}{(s^2+b^2)^2}$
$t \cos bt$	$\frac{s^2-b^2}{(s^2+b^2)^2}$
$e^{-at} \sin bt$	$\frac{b}{(s+a)^2+b^2}$
$e^{-at} \cos bt$	$\frac{s+a}{(s+a)^2+b^2}$
$1 - e^{-at} \left(\cos bt + \frac{a}{b} \sin bt \right)$	$\frac{a^2+b^2}{s[(s+a)^2+b^2]}$
$A = 1 - e^{-aT} \left[\cos bT + \left(\frac{a}{b} \right) \sin bT \right]$	
$B = e^{-2aT} + e^{-aT} \left[\left(\frac{a}{b} \right) \sin bT - \cos bT \right]$	