Exam Number: _____

Mathematics Qualifying Examination
Department of Mechanical Engineering

August 2012

Open Book: Only One Book Allowed, No Notes

Answer All Questions
All Questions Have Equal Weight

Time: 3 hours

Prepared by: C.J. Radcliffe and N. Mueller

Revision: 5/3/2012

1. Linear Algebra: Show all work – no calculator solutions

   a) Find the inverse of the matrix $\mathbf{A}$

$$\mathbf{A} = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 2 & 3 & 5 \\ 0 & 5 & 4 & 0 \end{bmatrix}$$

   b) Find the solution vector $\mathbf{x}$ to the set of equations

$$\mathbf{Ax} = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 2 & 3 & 5 \\ 0 & 5 & 4 & 0 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

   c) Find the eigenvalues and eigenvectors of the matrix $\mathbf{B}$.

$$\mathbf{B} = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$$

Solution(s):

2. Ordinary Differential Equations:
   a) Use the Laplace Transform to find the solution to the ODE

$$\frac{d^2y}{dt^2} + 4y = 3,\ y(0) = 1,\quad \frac{dy(0)}{dt} = 0$$

   b) Find the solution $y_1(t)$ for the system ordinary differential equations

$$\frac{d}{dt}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \text{ where at } t = 0,\ \begin{bmatrix} y_1(0) \\ y_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Note: A Laplace transform table is attached to this exam

Solution(s):

3. Infinite Series: Show all work – no calculator solutions

   a) Using a Taylor series approximation (Taylor's theorem) find the value of f(6) given that f(4)=125, f'(4)=74, f''(4)=30, f'''(4)=6 and all other higher derivatives of f(x) at x=4 are zero.

   b) Find the Fourier coefficients and Fourier series of the function defined by $f(x) = |x| \quad for -1 \le x < 1 \quad and \quad f(x+2) = f(x) \quad for\ all\ x$

Solution(s):

4. Partial Differential Equations: Show all work – no calculator solutions

Solve $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$ with y(0,t)=0 , y(L,t)=0 , y(x,0)=f(x) , $\frac{\partial y}{\partial t}$(x,0)=g(x)

Solution(s):

Laplace Transform Table:

Time function $e(t)$	Laplace transform $E(s)$
$u(t)$	$\frac{1}{s}$
t	$\frac{1}{s^2}$
$\frac{t^2}{2}$	$\frac{1}{s^3}$
t^{k-1}	$\frac{(k-1)!}{s^k}$
e^{-at}	$\frac{1}{s+a}$
te^{-at}	$\frac{1}{(s+a)^2}$
$t^k e^{-at}$	$\frac{(k-1)!}{(s+a)^k}$
$1-e^{-at}$	$\frac{a}{s(s+a)}$
$t-\frac{1-e^{-at}}{a}$	$\frac{a}{s^2(s+a)}$
$1-(1+at)e^{-at}$	$\frac{a^2}{s(s+a)^2}$
$e^{-at}-e^{-bt}$	$\frac{b-a}{(s+a)(s+b)}$
$\sin bt$	$\frac{b}{s^2+b^2}$
$\cos bt$	$\frac{s}{s^2+b^2}$
$t \sin bt$	$\frac{2bs}{(s^2+b^2)^2}$
$t \cos bt$	$\frac{s^2-b^2}{(s^2+b^2)^2}$
$e^{-at}\sin bt$	$\frac{b}{(s+a)^2+b^2}$
$e^{-at}\cos bt$	$\frac{s+a}{(s+a)^2+b^2}$
$1-e^{-at}\left(\cos bt+\frac{a}{b}\sin bt\right)$	$\frac{a^2+b^2}{s[(s+a)^2+b^2]}$

$$A = 1-e^{-aT}\left[\cos bT+\left(\frac{a}{b}\right)\sin bT\right]$$

$$B = e^{-2aT}+e^{-aT}\left[\left(\frac{a}{b}\right)\sin bT-\cos bT\right]$$