Code Number :.....

MATHEMATICS QUALIFYING EXAM

August 2007

OPEN BOOK (only one book allowed)

Answer all questions

All questions have equal weight

TIME: 3.0 hrs

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#1) How many linearly independent vectors are there in the following matrix

$$\mathbf{A}=\begin{pmatrix} 2 & 1 & -3 & 4 \\ 2 & 4 & -2 & 5 \\ 0 & 3 & 1 & 3 \\ 2 & 1 & -3 & 2 \end{pmatrix}$$

#2) Find one eigenvalue of each of the matrices below

$$\mathbf{A}=\begin{pmatrix}9 & 9 & 12\\ 9 & 10 & 11\\ 10 & 10 & 10\end{pmatrix},\ \mathbf{B}=\begin{pmatrix}2 & 2 & 1\\ 3 & 1 & 2\\ 0 & 2 & 2\end{pmatrix}$$ (hint: refer to Markov matrices)

#3) Solve the following initial value problem by means of the Laplace transform.

$y''''-16y=0;$

$y(0)=y'(0)=y''(0)=0, y'''(0)=3$

#4) Obtain the general solution to the following system of differential equations (primes denote $\frac{d()}{dt}$)

$x' = x + 4y$

$y' = x + y$

#5) Verify the divergence theorem by working out $\int_V \nabla \cdot \mathbf{v} dV$ and $\int_S \mathbf{n} \cdot \mathbf{v} dA$ and show that the results are equal for the vector field $\mathbf{v} = x\mathbf{i} + 2y\mathbf{j}$ where V is a cube $|x| \le 1, |y| \le 1, |z| \le 1$.

#6) Find the general and particular solutions of the differential equation

$y''' - y'' - 3x^2 + e^{2x} = 0$