Student ID ______________________

Department of Mechanical Engineering
Michigan State University

Ph.D. Qualifying Exam in Fluid Mechanics

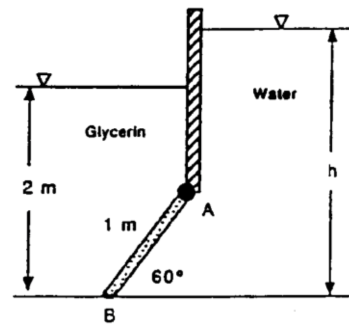
- Closed book and Notes, Some basic equations are provided on an attached information sheet.
- Answer all questions.
- All questions have the same weighting.

Exam prepared by

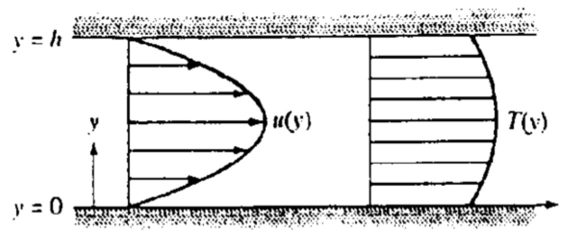
Professor F. Jaberi
Professor M. Zayernouri

January 2022

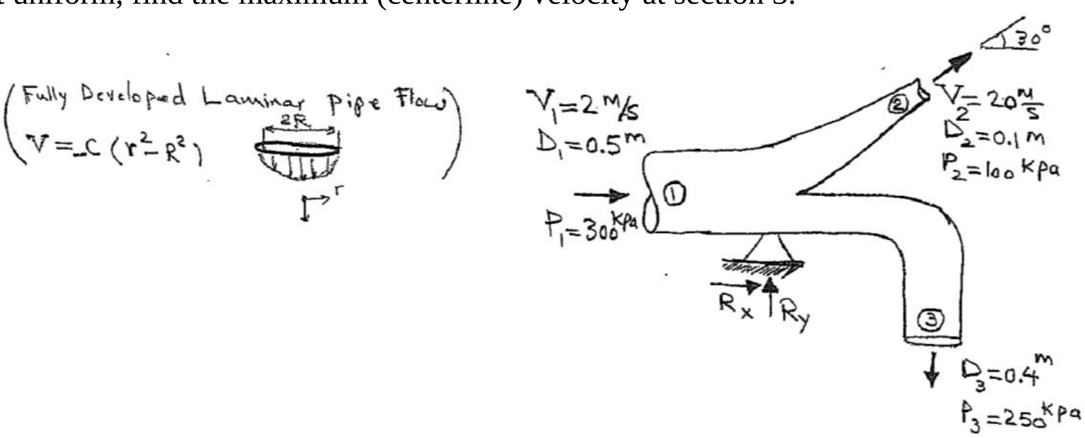
Problem 1: Gate **AB** is made of a homogeneous material, has a total mass of 180 kg, is 1.0m long, 1.2 m wide into the paper and is resting on the smooth bottom B. Let density of glycerin to be 1260 kg/m^3 and that of the water to be 998 kg/m^3 and the gravitational acceleration to be **g**=9.81 m/s^2. Find for what depth **h** the force at point **B** will be zero.



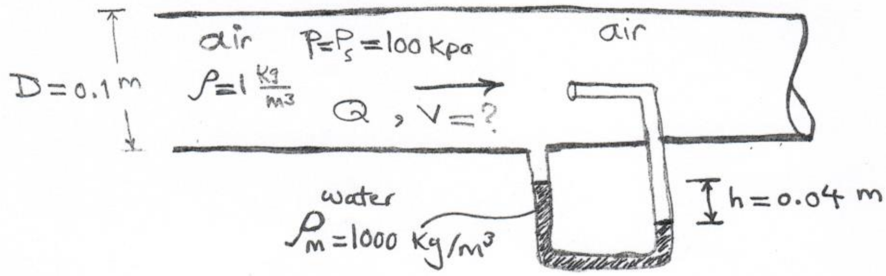
Problem 2: **(a)** Find the velocity profile **u(y)** for the steady-state laminar flow between the two parallel plates as shown in the figure. **(b)** If the wall temperature (both at the top and bottom walls) is fixed at **T$_w$** find the temperature distribution between the two walls, **T(y)** by using the energy equation. [Write all necessary assumptions.]



Problem 3: Water with density of 1000 kg/m^3 flows through the following system. By assuming uniform velocity at all sections, **(a)** calculate the mass flow rate of water at section 3, **(b)** Find reaction forces **R$_x$** and **R$_y$**. **(c)** By assuming the velocity at section 3 to be fully developed laminar and not uniform, find the maximum (centerline) velocity at section 3.



Problem (4) For the air duct shown below calculate: **(a)** Total pressure **P$_t$** , and dynamic pressure **P$_d$**, **(b)** The air flow velocity **V** and flow rate **Q**. The flow can be assumed to be incompressible and steady. Ignore the air weight. The air (static) pressure in the duct is 100Kpa. Gravitational acceleration is g=9.81 m/s^2.



Formula Sheet

Bernoulli Equation:

Bernoulli Equation between two points ① and ② on a streamline for a steady, incompressible flow:

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + g z_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + g z_2$$

Integral equations:

Mass : $0 = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho \vec{V}.d\vec{A}$

Momentum: $\vec{F} = \vec{F}_s + \vec{F}_B = \frac{\partial}{\partial t}\int_{CV} \vec{V}\rho d\forall + \int_{CS} \vec{V}\rho\vec{V}.d\vec{A}$

Differential Equations - Continuity

Rectangular Coordinates (x, y, z):

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

Differential Equations – Momentum for Incompressible Flows

Rectangular Coordinates (x, y, z):

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = \mu\left[\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right] - \frac{\partial p}{\partial x} + \rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = \mu\left[\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right] - \frac{\partial p}{\partial y} + \rho g_y$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = \mu\left[\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right] - \frac{\partial p}{\partial z} + \rho g_z$$

Differential Equations – Energy for Incompressible Flows:

$$\rho\left(\frac{\partial h}{\partial t} + V_x\frac{\partial h}{\partial x} + V_y\frac{\partial h}{\partial y} + V_z\frac{\partial h}{\partial z}\right) = K\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right)$$
$$+ \mu\left[2\left(\frac{\partial V_x}{\partial x}\right)^2 + 2\left(\frac{\partial V_y}{\partial y}\right)^2 + 2\left(\frac{\partial V_z}{\partial z}\right)^2 + \left(\frac{\partial V_x}{\partial y} + \frac{\partial V_y}{\partial x}\right)^2 + \left(\frac{\partial V_x}{\partial z} + \frac{\partial V_z}{\partial x}\right)^2\right.$$
$$\left. + \left(\frac{\partial V_y}{\partial z} + \frac{\partial V_z}{\partial y}\right)^2\right]$$

Where $h = c_p T$