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Ph.D. Qualifying Exam in Fluid Mechanics

- Closed book and Notes, Some basic equations are provided on an attached information sheet.
- Answer all questions.
- All questions have the same weighting.

Exam prepared by

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Problem 1: The bottom of a river has a 4-m-high bump that approximates a Rankine half-body, as in the figure below. The equation describing the bump surface geometry, in polar coordinates, is given as follows:

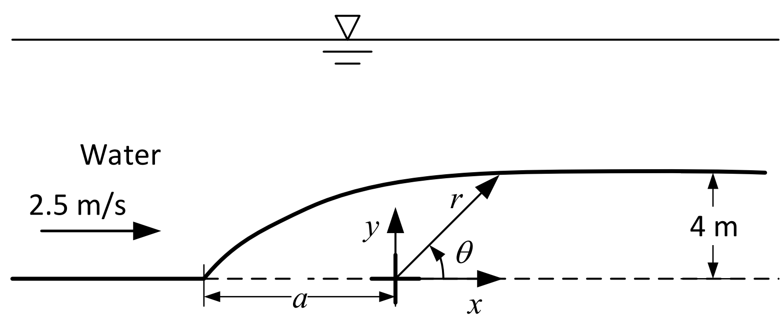
$$\frac{r}{a} = \frac{(\pi - \theta)}{\sin(\theta)}$$

The pressure on the flat bottom far ahead of the bump is 130 kPa, and the river velocity is $U_\infty = 2.5\ m/s$. Assuming steady, incompressible, inviscid flow, the streamwise and the cross-stream velocity components are given by, respectively:

$$\frac{v_x}{U_\infty} = 1 + \left(\frac{a}{r}\right)\cos(\theta)$$

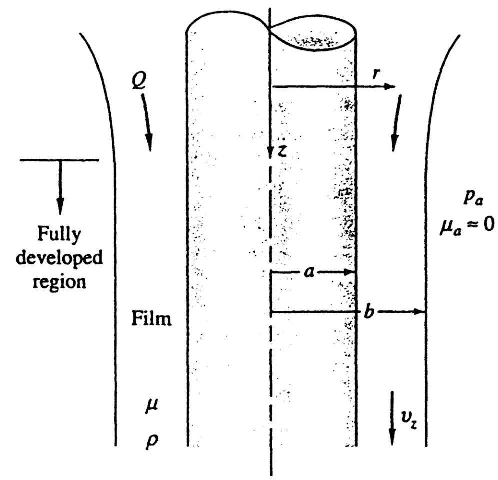
$$\frac{v_y}{U_\infty} = \left(\frac{a}{r}\right)\sin(\theta)$$

Determine the pressure at a point on the bump that is 2 m above the flat bottom. Take the water density to be $1000\ kg/m^3$ and the acceleration of gravity to be $9.81\ m/s^2$.



Problem 2: Consider a viscous film of liquid draining uniformly down the side of a vertical rod of radius a, as in the figure below. At some distance down the rod, the film will approach a terminal, or *fully developed*, draining flow of constant outer radius b, with $v_z = v_z(r), v_\theta(r) = v_r = 0$. Assuming that the atmosphere offers no shear resistance to the film motion, do the following:

1. Derive a differential equation for v_z, state the proper boundary conditions, and solve for the film velocity distribution.
2. How does the film radius b relate to the total film volume flow rate Q?

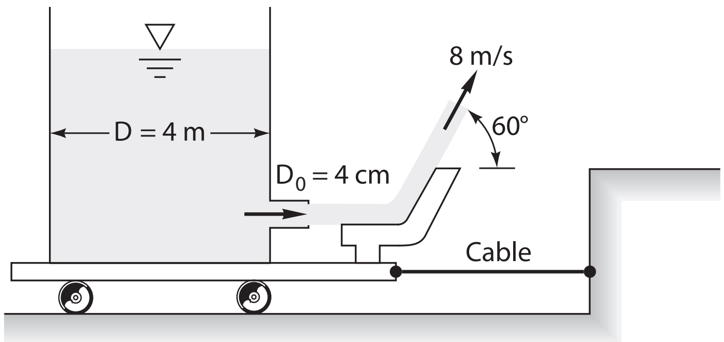


Problem 3: A thin elastic wire is placed between rigid supports. A fluid flows past the wire, and it is desired to study the static deflection, δ, at the center of the wire due to the fluid drag. Assume that δ depends on the wire length l, the wire diameter d, the fluid density ρ, the fluid viscosity μ, the fluid speed V, and the modulus of elasticity of the wire material E (note that the modulus of elasticity has dimensions of stress).
Consider a power line suspended between two towers and exposed to a 20 m/s wind. The diameter of the power line is $d = 1$ cm and its length is $L = 100$ m. If you want to use a 1:5 scale model in a water tunnel, determine the water speed in the tunnel to ensure that the model represents the conditions of the real power line.

Consider the following physical properties:

$\rho_{\text{water}} = 1{,}000$ kg/m^3
$\rho_{\text{air}} = 1$ kg/m^3
$\mu_{\text{water}} = 1 \times 10^{-3}$ Pa·s
$\mu_{\text{air}} = 18 \times 10^{-6}$ Pa·s

Problem 4. The water tank in the figure stands on a frictionless cart and feeds a jet of diameter $d = 4$ cm and velocity $V_{\mathrm{j}} = 8$ m/s, which is deflected 60° by a vane. Using control volume analysis, compute the tension in the supporting cable (consider $\rho_{\text{water}} = 1{,}000$ kg/m^3).



Possibly useful information and formulas:

- Bernoulli's Equation: $\frac{p_1}{\rho g}+z_1+\frac{V_1^2}{2g}=\frac{p_2}{\rho g}+z_2+\frac{V_2^2}{2g}$

- Pressure: p has units N/m^2 = Pascals. One atmosphere = *1.01325 E+5 Pa*. Force: $1\ N = 1\ kg\text{-}m/s^2$. Newton: $1\ N = 1\ kg\text{-}m/s^2$; Joule (Work) = *N-m*; Watt (Power) = N-m/s. *1 ft = 0.3048 m; 1 in = 2.54 cm; 1 mile = 5280 ft.* $1\ m^3 = 10^3 L$.

Integral equations:

Mass : $$0=\frac{\partial}{\partial t}\int_{CV}\rho d\forall+\int_{CS}\rho\vec{V}.d\vec{A}$$

Momentum: $$\vec{F}=\vec{F}_s+\vec{F}_B=\frac{\partial}{\partial t}\int_{CV}\vec{V}\rho d\forall+\int_{CS}\vec{V}\rho\vec{V}.d\vec{A}$$

Angular momentum: $$\sum(\vec{r}\times\vec{F})=\frac{\partial}{\partial t}\int_{CV}(\vec{r}\times\vec{V})\rho d\forall+\int_{CS}(\vec{r}\times\vec{V})\rho\vec{V}\cdot d\vec{A}$$

Differential Equations - Continuity

Rectangular Coordinates (x, y, z):

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x}(\rho v_x)+\frac{\partial}{\partial y}(\rho v_y)+\frac{\partial}{\partial z}(\rho v_z)=0$$

Cylindrical Coordinates (r, θ, z):

$$\frac{\partial\rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r)+\frac{1}{r}\frac{\partial}{\partial\theta}(\rho v_\theta)+\frac{\partial}{\partial z}(\rho v_z)=0$$

Differential Equations – Momentums for Incompressible, Constant Viscosity (μ)

Rectangular Coordinates (x, y, z):

$$\rho\left(\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}\right)=\mu\left[\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right]-\frac{\partial p}{\partial x}+\rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t}+v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}+v_z\frac{\partial v_y}{\partial z}\right)=\mu\left[\frac{\partial^2 v_y}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}+\frac{\partial^2 v_y}{\partial z^2}\right]-\frac{\partial p}{\partial y}+\rho g_y$$

$$\rho\left(\frac{\partial v_z}{\partial t}+v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z}\right)=\mu\left[\frac{\partial^2 v_z}{\partial x^2}+\frac{\partial^2 v_z}{\partial y^2}+\frac{\partial^2 v_z}{\partial z^2}\right]-\frac{\partial p}{\partial z}+\rho g_z$$

Cylindrical Coordinates (r, θ, z):

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial\theta}-\frac{v_\theta^2}{r}+v_z\frac{\partial v_r}{\partial z}\right)=\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right)+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial\theta^2}+\frac{\partial^2 v_r}{\partial z^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial\theta}\right]-\frac{\partial p}{\partial r}+\rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial\theta}+\frac{v_r v_\theta}{r}+v_z\frac{\partial v_\theta}{\partial z}\right)=\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right)+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial\theta^2}+\frac{\partial^2 v_\theta}{\partial z^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial\theta}\right]-\frac{1}{r}\frac{\partial p}{\partial\theta}+\rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial\theta}+v_z\frac{\partial v_z}{\partial z}\right)=\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial\theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right]-\frac{\partial p}{\partial z}+\rho g_z$$