

Q1 : A strain rosette contains three RSG positioned at  
a, b, c corresponding to  $0^\circ$ ,  $45^\circ$  and  $90^\circ$  resp.  
The reading obtained from the three gages are

$$\epsilon_a = -100 \mu\epsilon$$

$$\epsilon_b = 50 \mu\epsilon$$

$$\epsilon_c = 100 \mu\epsilon$$

Determine

- (A) the in plane principal strains
- (B) the maximum in plane shear strain
- (C) orientation of the principal axes

$$SI \cdot A \quad \epsilon_a = \epsilon_x \cos^2 0 + \epsilon_y \sin^2 0 + \gamma_{xy} \sin 0 \cos 0$$

$$\Rightarrow \epsilon_a = \boxed{\epsilon_x = -100 \mu\epsilon}$$

$$\epsilon_b = \epsilon_x \cos^2 45^\circ + \epsilon_y \sin^2 45^\circ + \gamma_{xy} \sin 45 \cos 45^\circ$$

$$50 = \frac{1}{2} (\epsilon_x + \epsilon_y + \gamma_{xy})$$

$$\epsilon_c = \epsilon_x \cos^2 90 + \epsilon_y \sin^2 90 + \gamma_{xy} \sin 90 \cos 90$$

$$\Rightarrow \epsilon_c = \boxed{\epsilon_y = 100 \mu\epsilon}$$

$$\Rightarrow \gamma_{xy} = 100 \mu\epsilon$$

$$\text{Principal strain} \quad \epsilon_{1,2} = \frac{1}{2} (\epsilon_x + \epsilon_y) \pm \frac{1}{2} \sqrt{(\epsilon_x - \epsilon_y)^2 + \gamma_{xy}^2}$$

$$= \pm \frac{1}{2} \sqrt{(-200)^2 + (100)^2}$$

$$= \pm \frac{1}{2} \sqrt{50,000} = \pm 111.803 \\ \approx \pm 112 \mu\epsilon$$

Max Shear strain,

$$(B) \quad \gamma_{max} = \pm (\epsilon_1 - \epsilon_2) = \pm 224 \mu\epsilon$$

(c) Orientation of principal axes

$$\tan 2\phi = \frac{\epsilon_{xy}}{\epsilon_x - \epsilon_y} = \frac{100}{-200}$$

$$2\phi = \tan^{-1} (-0.5)$$

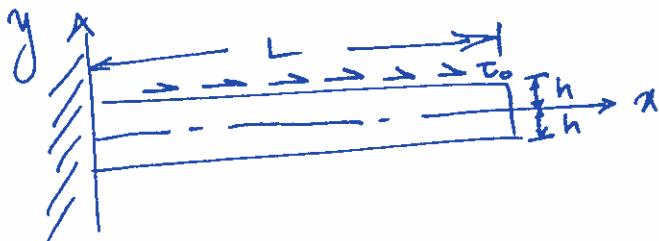
$$\phi = \frac{1}{2} \tan^{-1} (-\frac{1}{2})$$

$$\boxed{\phi = -13.28^\circ}$$

Q2 : A thin Cantilever shown in Figure below, is subjected to uniform shearing stress  $\tau_0$  along its upper surface ( $y = +h$ ) while other surfaces are stress free. Determine whether the Airy Stress function

$$\phi = \frac{1}{4} \tau_0 \left( xy - \frac{x^2 y^2}{h} - \frac{x^3 y^3}{h^2} + \frac{Ly^2}{h} + \frac{Ly^3}{h^2} \right)$$

Satisfies the required conditions for this problem.



For  $\phi$  to be a valid solution,

S2 : (A) The given  $\phi$  should satisfy the bi-harmonic equation

(B) All the Boundary Conditions need to be satisfied

(A)  $\nabla^4 \phi = 0$

$$\frac{\partial^4 \phi}{\partial x^4} + 2 \frac{\partial^4 \phi}{\partial x^2 \partial y^2} + \frac{\partial^4 \phi}{\partial y^4} = 0$$

$$\phi = \frac{T_0}{4} \left\{ xy - \frac{xy^2}{h} - \frac{xy^3}{h^2} + \frac{Ly^2}{h} + \frac{Ly^3}{h^2} \right\}$$

There are no 4<sup>th</sup> powers of  $x$  &  $y$

$$\Rightarrow \frac{\partial^4 \phi}{\partial x^4} = 0, \quad \frac{\partial^4 \phi}{\partial y^4} = 0$$

$$\frac{\partial^2 \phi}{\partial x^2} = 0 \quad \left\{ \text{no 2nd powers of } x \right\}$$

$$\Rightarrow \frac{\partial^4 \phi}{\partial x^2 \partial y^2} = 0$$

$\boxed{\nabla^4 \phi = 0}$  is satisfied &  
this might be a possible function.

(B) Boundary conditions,

$$\textcircled{2} \quad y = +h, \quad \sigma_y = 0 \\ \tau_{xy} = \tau_0$$

$$\textcircled{3} \quad y = -h, \quad \sigma_y = 0 \\ \tau_{xy} = 0$$

$$\textcircled{4} \quad x = +L, \quad \sigma_x = 0 \\ \tau_{xy} = 0$$

$$\text{from } \phi, \quad \sigma_x = \frac{\partial^2 \phi}{\partial y^2} \quad \sigma_y = \frac{\partial^2 \phi}{\partial x^2}$$

$$\sigma_{xy} = \tau_{xy} = -\frac{\partial^2 \phi}{\partial x \partial y}$$

$$\Rightarrow \sigma_x = \frac{\tau_0}{4h} \left\{ 2(L-x) + \frac{6y}{h} (L-x) \right\}$$

$$= \frac{\tau_0}{4h} \left( 2 + \frac{6y}{h} \right) (L-x)$$

$$\sigma_y = 0 \quad \dots \quad (\text{no 2nd power of } x \text{ in } \phi)$$

$$T_{xy} = -\frac{T_0}{4} \left\{ 1 - \frac{2y}{h} - \frac{3y^2}{h^2} \right\}$$

$$\Rightarrow @ y = h, \quad \sigma_y = 0 \quad \checkmark \text{ satisfied}$$

$$T_{xy} = -\frac{T_0}{4} \left\{ 1 - 2 - 3 \right\}$$

$$T_{xy} = T_0 \quad \checkmark \text{ satisfied}$$

$$@ y = -h, \quad \sigma_y = 0 \quad \checkmark \text{ satisfied}$$

$$T_{xy} = -\frac{T_0}{4} \left\{ 1 + 2 - 3 \right\}$$

$$T_{xy} = 0 \quad \checkmark \text{ satisfied}$$

$$@ x = L, \quad \sigma_x = \frac{T_0}{4h} \left( 2 + \frac{6y}{h} \right) (L-L)$$

$$\sigma_x = 0 \quad \checkmark \text{ satisfied}$$

$$T_{xy} = -\frac{T_0}{4} \left\{ 1 - \frac{2y}{h} - \frac{3y^2}{h^2} \right\}$$

$$\boxed{T_{xy} \neq 0} \quad \text{Not Satisfied.}$$

$\Rightarrow$  We can not use this  $\phi$  as solution because it can not satisfy all Boundary conditions.

- 3) A circular shaft of radius  $c$  is subjected to torsion. The material is made of an aluminum alloy which exhibits an elastic, power-hardening curve such that  $\tau = G\gamma$  below yielding ( $\tau \leq \tau_o$ ) and  $\tau = H\gamma^n$  above yielding ( $\tau \geq \tau_o$ ). Derive an equation that will relate the torque  $T$  with the strain at  $r=c$ ,  $\gamma_c$  under an elastic-plastic deformation.

Answer: Under the elasto-plastic deformation, the elasto-plastic boundary occurs at  $r=r_b$ .

$$\gamma_o = \frac{\tau_o}{G}, \frac{\gamma_o}{r_b} = \frac{\gamma_c}{c}, r_b = \frac{\tau_o c}{G\gamma_c}$$

$$(0 \leq r \leq r_b)$$

$$\frac{\tau}{Gr} = \frac{\tau_o}{G\tau_b}, \quad \tau = \frac{\tau_o r}{r_b}$$

$$(r_b \leq r \leq c)$$

$$\frac{\gamma}{r} = \frac{\gamma_c}{c}, \quad \gamma = \frac{\gamma_c r}{r_b}, \quad \tau = H \left( \frac{\gamma_c r}{c} \right)^n$$

$$T = 2\pi \left[ \int_o^{r_b} \tau r^2 dr + \int_{r_b}^c \tau r^2 dr \right]$$

$$\frac{T}{2\pi} = \frac{\tau_o}{r_b} \int_o^{r_b} r^3 dr + H \left( \frac{\gamma_c}{c} \right)^n \int_{r_b}^c r^{2+n} dr$$

$$\frac{T}{2\pi} = \frac{\tau_o}{r_b} \left( \frac{r_b^4}{4} \right) + H \left( \frac{\gamma_c}{c} \right)^n \frac{c^{3+n} - r_b^{3+n}}{3+n}$$

$$\frac{T}{2\pi} = \frac{\tau_o}{4} \left( \frac{\tau_o c}{G\gamma_c} \right)^3 + H \gamma_c^n \frac{c^3}{3+n} - \frac{H}{3+n} \left( \frac{\gamma_c}{c} \right)^n \left( \frac{\tau_o c}{G\gamma_c} \right)^{3+n}$$

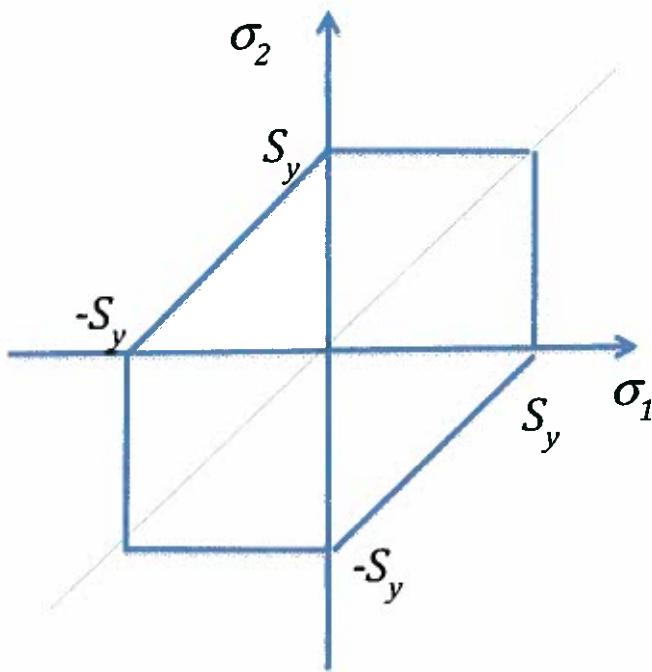
$$\frac{T}{2\pi c^3} = \frac{G\gamma_o^4}{4\gamma_c^3} + \frac{H\gamma_c^n}{3+n} - \frac{H}{3+n} \frac{\gamma_o^{3+n}}{\gamma_c^3}$$

$$T = \frac{2\pi c^3}{3+n} \left[ \frac{G\gamma_o^4 (3+n)}{4\gamma_c^3} + H\gamma_c^n - \frac{H\gamma_o^n \gamma_o}{\gamma_c^3} \right]$$

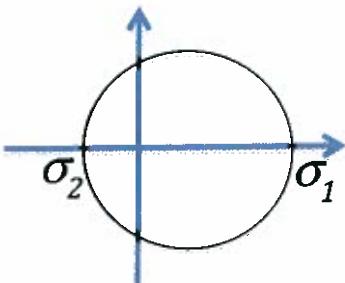
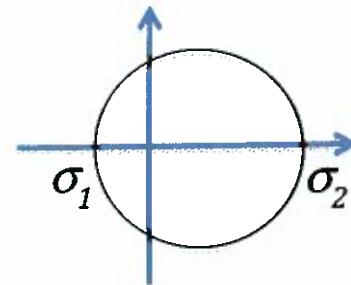
$$\text{where } H\gamma_o^n = \tau_o = G\gamma_o$$

$$T = \frac{2\pi c^3}{n+3} \left[ \frac{G\gamma_o^4 (n-1)}{4\gamma_c^3} + H\gamma_c^n \right]$$

- 4) Tresca Criterion for yielding stated that yield occurs when the maximum shear stress on any plane reaches a critical value. The yield envelope for plane stress shown below. Please show how the envelope can be attained.



When  $\sigma_1$  and  $\sigma_2$  have different signs

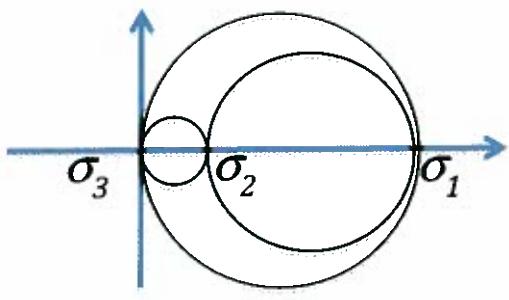


$$|\sigma_1 - \sigma_2| = S_y \quad |\sigma_1| = S_y \quad \text{or} \quad |\sigma_2| = S_y \quad \sigma_1 > 0$$

$$\sigma_1 - \sigma_2 = S_y \quad \sigma_2 - \sigma_1 = S_y$$

$$\sigma_2 > 0 \quad \sigma_2 = \sigma_1 - S_y \quad \sigma_2 = \sigma_1 + S_y$$

When  $\sigma_1$  and  $\sigma_2$  have the same sign



$$|\sigma_1| = S_y \quad \text{or} \quad |\sigma_2| = S_y$$