Department of Mechanical Engineering
Michigan State University

Ph.D. Qualifying Exam in Intermediate Solid Mechanics

- Open book. Two books are allowed.
- **In order to receive full credit, you must show all work.** Use decimals, not fractions and box your final answer. Do not forget units!
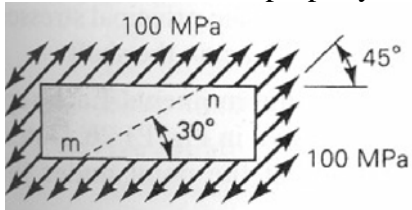
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1. The stress acting uniformly over the sides of a rectangular block is shown below. Calculate the stress components on planes parallel and perpendicular to the plane on *mn*. Show the results on properly oriented element.

2. Consider a rectangular plate, made of four lines, x=0, x=a, y=0, and y=b of thickness t. (a) Determine the stresses σ_x, σ_y, and τ_{xy} for the stress function $\Phi=px^3y$, where p is a constant. (b) Draw a sketch showing the boundary stresses on the plate. (c) Find the resultant normal and shearing boundary forces (P_x, P_y, V_x, and V_y) along all edges of the plate.
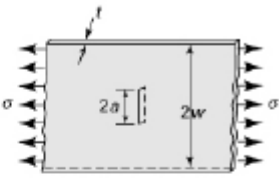
3. Two cylindrical shafts are to be subjected to pure torsion. One has a hollow ($r_i=b$, $r_0=c$) and the other has a solid ($r_0=a$) cross-section. The two shafts have the same length and cross-sectional area, and are made of the same material. Determine the largest torques that may be applied to the shafts for c=1.4b (a) for an allowable stress τ_a; and (b) for an allowable angle of twist θ_a.

4. A plate is made of Al 2024-T851. The plate has a length and width ratio >5 and hence can be approximated as infinite long. The plate has a width = 0.15m and thickness =0.03m. The NDT technique can detect a crack when its length is greater than 0.01m. For edge crack, what is the maximum allowable axial load the plate can take without sudden fracture? Will the plate yield before fracture?

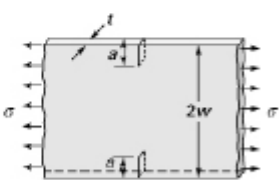TABLE 4.2. *Geometry Factors λ for Initial Crack Shapes*

Case A Tension of a long plate with central crack

a/w	λ
0.1	1.01
0.2	1.03
0.3	1.06
0.4	1.11
0.5	1.19
0.6	1.30
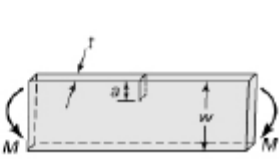
Case B Tension of a long plate with edge crack

a/w	λ
0 ($w \to \infty$)	1.12
0.2	1.37
0.4	2.11
0.5	2.83

Case C Tension of a long plate with double edge cracks

a/w	λ
0 ($w \to \infty$)	1.12
0.2	1.12
0.4	1.14
0.5	1.15
0.6	1.22

Case D Pure bending of a beam with edge crack

a/w	λ
0.1	1.02
0.2	1.06
0.3	1.16
0.4	1.32
0.5	1.62
0.6	2.10

Metals	σ_{yp}		K_c	
	MPa	(ksi)	$MPa\sqrt{m}$	$(ksi\sqrt{in.})$
Aluminum				
2024-T851	444	(64.4)	23	(20.9)
7075-T7351	392	(56.9)	31	(28.2)

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