Student Code Number:_____

Ph.D. Qualifying Exam

Intermediate Solid Mechanics
Spring 2010

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Directions: Closed Book and Notes
You may use a one page formula sheet.

Answer all four questions

All questions have equal weight

Time: 3.0 hrs.

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- Take any required property from your book, approximate values if necessary.
- If you make any assumption to reach a solution state it clearly

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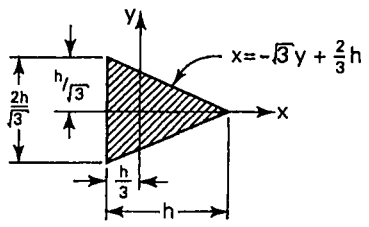
1. Consider the torsion problem in a bar with equilateral triangular cross-section as shown. Assuming the Prandtl stress function Φ to be of the form:

$$\Phi = k(x - \sqrt{3}y - \tfrac{2}{3}h)(x + \sqrt{3}y - \tfrac{2}{3}h)(x + \tfrac{1}{3}h)$$

   (a) Find k in terms of shear modulus G and angle of twist per unit length θ.
   (b) Find maximum shear stress in terms of torque T
   (c) Derive an expression for the torsional rigidity $C=T/\theta$.



2. Derive the equation of equilibrium for thick-walled cylinder from the equation of equilibrium in cylindrical coordinates, which states

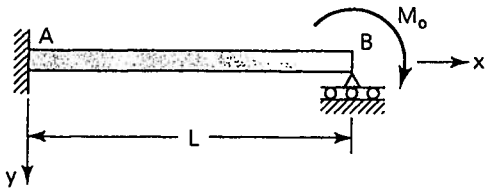
$$\frac{\partial \sigma_r}{\partial r}+\frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta}+\frac{\partial \tau_{zr}}{\partial z}+\frac{\sigma_r-\sigma_\theta}{r}=0$$

$$\frac{\partial \tau_{r\theta}}{\partial r}+\frac{1}{r}\frac{\partial \sigma_\theta}{\partial \theta}+\frac{\partial \tau_{\theta z}}{\partial z}+\frac{2\tau_{r\theta}}{r}=0$$

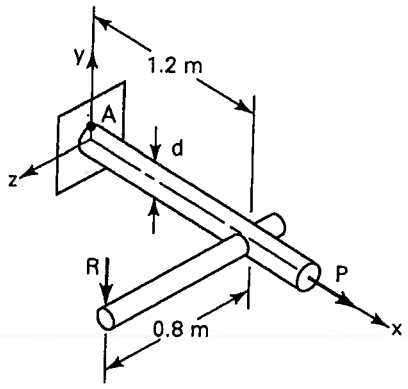
$$\frac{\partial \tau_{zr}}{\partial r}+\frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta}+\frac{\partial \sigma_z}{\partial z}+\frac{\tau_{zr}}{r}=0$$

The result should be a second-order differential equation in terms of the radial displacement, u. Use the definition of the strains in cylindrical coordinates and linear elastic constitutive equation. State all the assumptions necessary.

3. A propped cantilever beam is subjected to a couple M_0 acting support B, as shown below. Derive the equation of the deflection curve and determine the reaction at the roller support.



4. A steel rod of diameter d=50 mm (yield strength, σ_y = 260 MPa) supports an axial load P = 50R and vertical load R acting at the end of an 0.8 m long arm as shown below. Given a factor of safety n = 2, compute the largest permissible value of R using the following criteria: (a) maximum shearing stress (Tresca) and (b) maximum distortion energy (von Mises).



ode	Q1	Q2	Q3	Q4	average	
1	100P	70P	90P	70P	82.5	P
2	85P	60B	70P	100P	78.75	P
3	20F	10F	50F	50F	32.5	F
4	90P	100P	100P	100P	97.5	P
5	50F	0F	70P	80P	50	F
6	60B	60B	80P	60B	65	P
7	20F	60B	90	60	59	F
8	35F	60B	60B	60B	53.75	F
9	20F	80P	60	70	57.5	F
10	10F	40F	70P	40F	40	F
11	50F	20F	100P	60P	57.5	F
12	70P	50F	100P	40F	65	P
13	100P	0F	40F	0F	35	F