

Student Code Number: \_\_\_\_\_

# **Ph.D. Qualifying Exam**

## **Heat Transfer**

**Fall 2009**

Professor George Zhu  
Professor Tonghun Lee

Directions: Open Book (only one book allowed) and closed notes

Answer all four questions

All questions have equal weight

Time: 3.0 hours

- .....
- Take any required property from your book, approximate values if necessary
  - If you make any assumptions to reach a solution, state it clearly
- .....

**Problem 1:** A metal ball (diameter  $D=10$  mm) has a very thin rubber coating (thickness = 0.3 mm). The ball is heated to  $100^\circ\text{C}$ . Then it is dropped into ice water at  $0^\circ\text{C}$  with a convection coefficient of  $h=25 \text{ W m}^{-2}\text{K}^{-1}$ . The metal ball has  $k_{\text{met}}=400 \text{ W m}^{-1}\text{K}^{-1}$  and  $\rho C_p=3.5 \times 10^6 \text{ J m}^{-3}\text{K}^{-1}$ .

The thin rubber coating has  $k_{\text{rub}}=0.1 \text{ W m}^{-1}\text{K}^{-1}$ . You can neglect the heat capacity of the rubber coating, which serves merely as thermal insulation for the ball.

a) Develop a thermal resistance circuit with four temperature nodes:

$T_1$ : mid-point temperature of the metal ball

$T_2$ : temperature at the metal rubber interface

$T_3$ : temperature at the rubber-water interface

$T_4$ : ice water temperature ( $0^\circ\text{C}$ )

Derive symbolic and numerical results for the three resistors between these four temperatures. Note that the resistor between  $T_1$  and  $T_2$  can only be approximated – just make a good guess for the value of this one.

b) Can you derive a symbolic expression for when it is appropriate to neglect temperature variations within the metal ball, i.e., when  $T_1$  and  $T_2$  are nearly identical? Is this criterion satisfied here?

c) Derive symbolically the time-dependence of the metal ball temperature as a function of time. Plot this result on a graph and find the characteristics time of the temperature drop ( $1/e$  decay of the original temperature)?

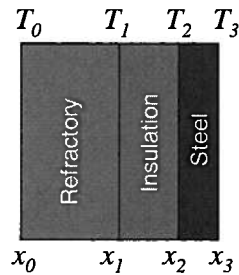
d) How long will it take for the ball temperature to reach  $20^\circ\text{C}$  ?

**Problem 2:** Consider radiative heat transfer between two parallel, infinite plates, with temperatures fixed at  $T_{\text{left}}=1000$  K and  $T_{\text{right}}=300$  K and each with emissivity of  $\epsilon=0.5$ . An intermediate plate with same emissivity helps reduce the heat flow.

- a) Draw the complete radiative resistive circuit, including all viewfactor and graybody resistances.
- b) Calculate the heat flux from one plate to the other, first in symbolic form, and then calculate the numbers.

What is the temperature of the middle plate (determine the result symbolically, then the numbers).

**Problem 3:** The wall of a furnace comprises three layers as shown in the figure below. The first layer is refractory (whose maximum allowable temperature is  $1400^{\circ}\text{C}$ ) while the second layer is insulation (whose maximum allowable temperature is  $1093^{\circ}\text{C}$ ). The third layer is a plate of 6.35 mm thickness of steel [thermal conductivity =  $45 \text{ W}/(\text{mK})$ ]. Assume the layers to be in very good thermal contact.

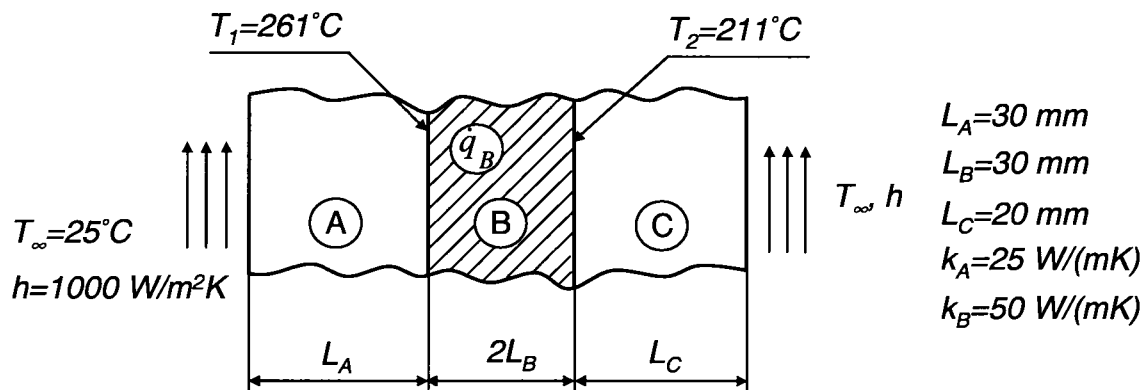


The temperature  $T_0$  on the inside of the refractory is  $1370^{\circ}\text{C}$ , while the temperature  $T_3$  on the outside of the steel plate is  $37.8^{\circ}\text{C}$ . The heat loss through the furnace wall is expected to be  $15800 \text{ W}/\text{m}^2$ . Determine the thickness of refractory and insulation that results in the minimum total thickness of the wall.

Given thermal conductivities in  $\text{W}/(\text{mK})$ :

Layer	$k$ at $37.8^{\circ}\text{C}$	$k$ at $1093.3^{\circ}\text{C}$
Refractory	3.12	6.23
Insulation	1.56	3.12

**Problem 4:** Consider a composite wall with outer surface exposed to a convection process while the inner wall experiences uniform heat generation, see figure below.



Assume the following:

- Steady-state and one dimensional heat transfer
- Negligible contact resistance at interfaces
- Uniform generation in B, and zero generation in A and C
- Constant properties

Find

- Heat loss balance equation on wall B
- Thermal circuits for A and C
- Volumetric heat generation  $\dot{q}_B$