Code Number :.............

HEAT TRANSFER QUALIFYING EXAM

January 2011

OPEN BOOK (only one book allowed)

Answer all questions

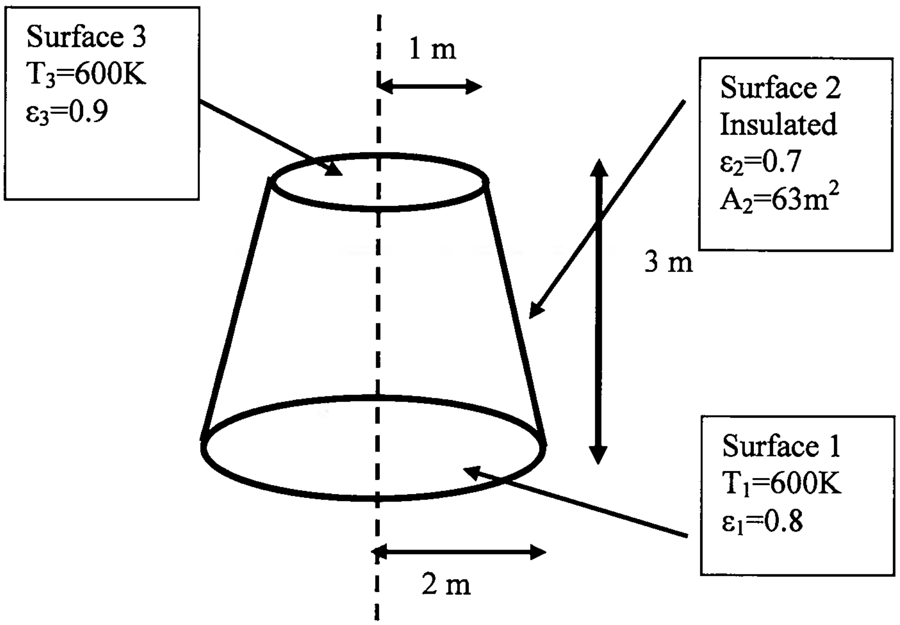
All questions have equal weight

TIME: 3.0 hrs

Prepared by : Profs. A. Bénard and N. Wright

Question #1)

The top and bottom surfaces of a truncated cone (the frustrum of a cone) shown below are kept at 500K and 1000K respectively. The side surface is perfectly insulated (surface 2). If all surfaces are gray and diffuse, find the system of equations that will allow you to compute the radiosity for each surface, ***do not*** solve the system of equations for brevity.



Question #2)

A submerged steel pipe ($k = 20$ W/m/°C) carries oil at 0.25 m^3/s across a river. The interior diameter of the pipe is 20 cm and the exterior diameter is 21 cm. At one bank of the river, the mean temperature of the oil is 90 °C and at the other bank it is 80 °C. The river flows at 3 m/s and the temperature of the water is 10 °C. Estimate the submerged length of the pipe.

Question #3)

A long steel bar with a cross-section of 10 cm by 30 cm is initially at a uniform temperature of 600 °C. The bar is plunged into a water bath that is at 100 °C. Calculate the temperature at the center of the bar after 20 min. Assume that the properties of the steel are $k = 20$ W/m/°C, $\rho = 8550$ kg/m^3, and $c_p = 550$ J/kg/°C. Further assume that the heat transfer coefficient is 5000 W/m^2/°C. (Note: the solution of a two-dimensional homogeneous problem can be written as the product of the appropriate one-dimensional solutions.)

Question #4)

A radiant oven (set on broiler) is at 900K. It is used to cook a thin round pizza of 0.75 cm – thick. The pizza is assumed to have the same properties as water and its surface area is much smaller than the heating element (view factor to the heating element is 1). If the pizza is taken from the refrigerator at 15°C, develop a mathematical model to find the time it takes for the pizza to reach 130°C. The pizza has a radius of 10 cm and an emissivity of 0.95. The surrounding air is at 20°C and the heat transfer coefficient is 10 W/m^{2}K. In developing your simple model, state under what conditions you could solve analytically the resulting equation e.g. is convection neglibible?

#1)

Find the view factors ⇒ fig 13.5 coaxial ∥ disks

$\frac{r_j}{L} = \frac{1}{3}$, $\frac{L}{r_i} = \frac{3}{2}$ ⇒ $\underline{F_{13} = 0.05}$

sum rule ⇒ $F_{12} + F_{13} = 1$ so $\underline{F_{12} = 0.95}$ &

reciprocity ⇒ $A_1 F_{13} = A_3 F_{31}$ & $A_2 F_{23} = A_3 F_{32}$

$$F_{31} = \frac{A_1}{A_3} F_{13} = \frac{4}{1} 0.05 = 0.2$$

$$F_{31} + F_{32} = 1 \text{ so } F_{32} = 0.8$$

$$\Rightarrow F_{23} = \frac{A_3}{A_2} F_{32} = \frac{1}{4} \cdot 0.8 = 0.2$$

$$F_{21} = F_{12} \cdot \frac{A_1}{A_2} = 0.95 \cdot \frac{3}{63}$$

$$F_{21} = 0.045$$

$$F_{21} + F_{22} + F_{23} = 1$$

$$F_{22} = 1 - 0.045 - 0.2$$

$$F_{22} = 0.755$$

$$A_2 = \pi\left[(4+1) + \sqrt{(4-1) + (3(4+1))^2}\right]$$

$$= \pi\left[5 + \sqrt{3 + 225}\right] = 63\ m^2$$

solve

$$E_{b1} \cdot \frac{\varepsilon_1}{1-\varepsilon_1} = J_1\left(1 - F_{11} + \frac{\varepsilon_1}{1-\varepsilon_1}\right) + J_2(-F_{12}) + J_3(-F_{13})$$

$$0 = J_1(-F_{21}) + J_2(1 - F_{22}) + J_3(-F_{23})$$

$$E_{b3} \cdot \frac{\varepsilon_3}{1-\varepsilon_3} = J_1(-F_{31}) + J_2(-F_{32}) + J_3\left(1 - F_{33} + \frac{\varepsilon_3}{1-\varepsilon_3}\right)$$

#2)

Qualifier - Jan 2010

$$q = \left(\dot{m} c_p \Delta T\right)_{oil} = \frac{(\Delta T_{lm})\,\pi L}{\frac{1}{h_i D_i} + \frac{\ln D_o/D_i}{2 k_s} + \frac{1}{h_o D_o}}$$

$D_o = 0.21\ m$; $D_i = 0.20$; $\dot{m} = \rho Q = 0.25\rho$ $\Delta T = 90 - 80 = 10°C$

$$\Delta T_{lm} = \frac{10}{\ln 80/70} = 74.9\ °C \qquad k_s = 20\ W/m{\cdot}K$$

For the oil, $Pr = 395$; $\nu = 30 \times 10^{-6}\ m^2/s$; $k = 0.138\ W/m{\cdot}K$; $c_p = 2161\ J/kg\,K$

$\rho = 847\ kg/m^3$

$$Re = \frac{4Q}{\pi D_i \nu} = \frac{(4)(0.25)}{(\pi)(0.20)(30\times10^{-6})} = 5.31 \times 10^4$$

$$Nu_D = 0.023\,(Re)^{0.8}(Pr)^{0.3} = 833 \longrightarrow h_i = 575\ W/m^2K$$

For the H_2O : $\rho = 987\ kg/m^3$, $\mu = 528 \times 10^{-6}\ N{\cdot}s/m^2$; $k = 0.645\ W/m\,K$

$Pr = 3.42$

$$Re = \frac{(3)(0.21)(987)}{528 \times 10^{-6}} = 1.2 \times 10^6$$

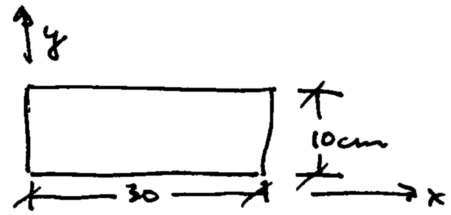
$$Nu = 0.3 + \frac{(0.62)(1.2\times10^6)^{1/2}(3.42)^{1/3}}{\left[1 + (0.4/3.42)^{2/3}\right]^{1/4}}\left[1 + \left(\frac{1.2\times10^6}{282000}\right)^{5/8}\right]^{4/5}$$

$$= 2560 \longrightarrow h = \frac{(2560)(0.645)}{0.21} = 7870\ W/m^2K$$

$$(0.25)(847)(2161)(10) = \frac{(74.9)(\pi)\,L}{\frac{1}{(0.2)575} + \frac{\ln 0.21/0.20}{(2)(20)} + \frac{1}{(0.21)(7870)}}$$

$\underline{\underline{L = 205\ m}}$

#31



Assume that $\Theta_o(x,y) = \Theta_o(x)\,\Theta_o(y)$

where $\Theta = \dfrac{T_o - T_\infty}{T_i - T_\infty}$

$$Bi_x = \frac{hL}{k} = \frac{(5000)(0.15)}{20} = 37.5$$

$$Bi_y = \frac{(5000)(0.05)}{20} = 12.5$$

Both too large for lumped capacitance

$$Fo_x = \frac{\alpha t}{L^2} = \frac{\left(\frac{20}{(8550)(550)}\right)(1200)}{(0.15)^2} = 0.227 > 0.2$$

$$Fo_y = (9)\,Fo_x = 2.04 > 0.2$$

Approx solution is okay for each x & y

$\therefore$ midpoint for in x & y $\quad \Theta_o^* = C_1 \exp(-\zeta_1^2 Fo)$

x. $\zeta_1 = 1.53$ (rad) $\quad C_1 = 1.2721$

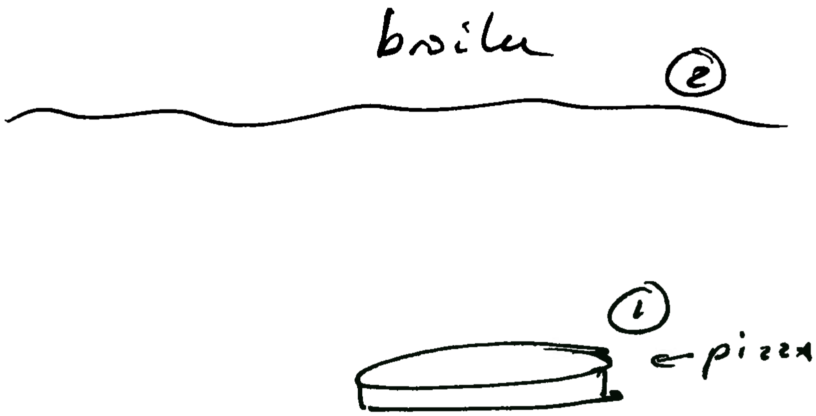
y. $\zeta_1 = 1.48$ (rad) $\quad C_1 = 1.27$

$$\Theta_o = \frac{T_o - T_\infty}{T_i - T_\infty} = \underbrace{(1.2721)}_{(1.2721)}\underbrace{\exp\left(-(1.53)^2(0.227)\right)}_{(0.588)}\underbrace{(1.27)}_{(1.27)}\underbrace{\exp\left(-(1.48)^2 (2.04)\right)}_{(0.0115)}$$

$= 0.0109$

$T_o = (0.0109)(600 - 100) + 100 = \underline{\underline{105\ ^\circ C}}$

Sketch of sol'n #4



Energy balance:

$$\dot{E}_{in} - \cancel{\dot{E}_{out}}^{\,0} + \cancel{\dot{E}_g}^{\,0} = \dot{E}_{st}$$ (neglect convection first)

$$q_{rad} = \dot{E}_{st}$$

$$q_{rad} = \varepsilon A_1 \sigma (T_2^4 - T_1^4)$$

$$\dot{E}_{st} \triangleq \rho V c_p \frac{dT_1}{dt}$$ (assume lumped)

separate variables & integrate $\rho \delta c_p \frac{dT_1}{dt} = \varepsilon A \sigma (T_2^4 - T_1^4)$

$$\int_0^t dt = \frac{\rho c_p \delta}{\varepsilon \sigma} \int_{298}^{403} \frac{dT}{(T_2^4 - T_1^4)}$$

use table & solve for t ...

→ if convection included, need numerical sol'n