HEAT TRANSFER QUALIFYING EXAM

January 2011

OPEN BOOK (one book allowed only)

Answer all four questions

All questions have equal weight

TIME: 3.0 hrs

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- Take any required property from your book, approximate values if necessary.
- If you make any assumption to reach a solution state it clearly
**Question # 1**

A cold beverage is contained in a tall cylindrical thermos bottle set upright on a table. The bottle has a height of 0.4 m and it consists of a glass flask of $D_1 = 0.07$ m surrounded by an aluminum housing of diameter $D_2 = 0.08$ m. The outer and inner surface of the housing are silver coated to provide emissivities of $\varepsilon_1 = \varepsilon_2 = 0.25$. If $T_1 = 5^\circ$C and $T_2 = 20^\circ$C, what is the rate of heat gain by the liquid? Use for properties of air in the gap: $k = 0.0284 W/mK$, $\nu = 23.74 \times 10^{-6} m^2/s$, $\alpha = 26.6 \times 10^{-6} m^2/s$, $Pr = 0.7$ for an air temperature of 300K.
The ceiling of an ice-skating rink should have a high reflectivity to avoid condensation on the ceiling. If condensation forms on the ceiling, water may drip on the ice causing bumps on the ice. The rink has a diameter of 50m and a height of 12m. The temperature of the ice and walls are -5°C and 15°C respectively. The air in the rink is also at 15°C. Neglect the convective losses to the outside air. If the ceiling is a diffuse-gray surface and the ice and walls can be approximated as black bodies, perform an energy balance on the ceiling to calculate the ceiling temperature. If the relative humidity of the rink air is 70% and the dew point is 9°C, will condensation occur if a coating is added that provides an emissivity of 0.05?

\[ T_{\text{ice}} = -5^\circ \text{C} \]
Question # 3: “Global Warming” by Convection: A flat iceberg drifts over the ocean, as it is driven by the wind that blows over the top. The iceberg may be modeled as a block of frozen fresh water at 0°C. The temperature of the surrounding seawater is 10°C and the relative velocity between it and the iceberg is 10 cm/s. The length of the iceberg in the direction of drift is $L = 100m$. The relative motion between the seawater and the flat bottom of the iceberg produces a boundary layer of length $L$. The 10°C temperature difference across the boundary layer drives a heat flux into the bottom of the iceberg. The heating effect causes the steady erosion (thinning) of the flat iceberg. If $H(t)$ is the instantaneous height of the ice slab, calculate the ice melting rate $dH/dt$ averaged over the swept length of the iceberg.
**Question # 4: Conduction Heat Transfer in a Circular Membrane:** Consider the steady-state heat equation $\nabla^2 T = -Q$. In cylindrical coordinates this equation takes the form

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} = -Q. \tag{1}
\]

Consider the case in which $\frac{\partial T}{\partial \theta} = 0$ and $Q = Q(r)$ in Eq. (1).

Examine two cases:

a) $Q = \text{constant} = 4$.

b) $Q = 36(1-2r)$ varies radially.

**NOTE:** In case (b) $Q(r)$ is positive for $0 < r < \frac{1}{2}$ and it is negative for $\frac{1}{2} < r < 1$.

Solve for the temperature $T(r)$ in cases (a) and (b) subject to the condition $T = 1$ at $r = 1$.

Why does this second order equation have only one BC?

Draw “side” views of $T(r)$ and comment especially on the heat term $Q(r)$ that produces it. Is $Q$ a source or a sink? Etc. In doing this you are strongly encouraged to make one plot ($Q(r)$) directly above the other ($T(r)$). You are also encouraged to point out details of the $T(r)$ plots such as maxima, minima, etc., their location (the value of $r$ at which they occur) and their physical significance.