Code	Num	ber	•	 •••

# **HEAT TRANSFER QUALIFYING EXAM**

## August 2007

### OPEN BOOK (only one book allowed) & CLOSED NOTES

Answer all four questions

All questions have equal weight

TIME: 3.0 hrs

Prepared by : F. Jaberi & I. Wichman

• Take any required property from your book, approximate values if necessary.

• If you make any assumption to reach a solution state it clearly ••••••••••••••

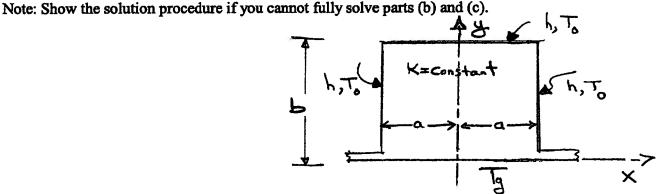
## Question #1

A jet engine combustor is cooled by water flowing in an annular jacket around it.

Fins running spirally in the annular jacket are cast integral with the combustion chamber wall and guide the cooling water. As an approximation, the fins may be considered straight and of rectangular cross section as shown below. In addition, the inside surface of the combustion chamber wall may be assumed to be at the temperature of combustion gasses  $T_g$ . The cooling water temperature  $T_0$  and the heat transfer coefficient, h may be considered constants.

- (a) Formulate the problem and write the appropriate differential equation and boundary conditions that describe the steady state temperature distribution in the fins.
- (b) Develop an expression for the steady-state temperature distribution T(x,y) in the fins by using the separation of variable method.

(c) Obtain an expression for the rate of heat dissipated by each fin per unit depth.



#### Problem 2:

Conduct an integral analysis of the thermal boundary layer near an isothermal wall when the fluid has a large Prandtl number,  $Pr = \delta/\delta_T >> 1$  (see the figure below).

- a) Show that the energy equation  $u\partial T/\partial x + v\partial T/\partial y = \alpha \partial^2 T/\partial y^2$  can be written for the Pr >> 1 case as  $\partial (uT)/\partial x + \partial (vT)/\partial y = \alpha \partial^2 T/\partial y^2$ .
- b) Integrate the energy equation across the thermal boundary layer and produce the integral energy equation

$$\frac{d}{dx} \int_{0}^{\delta_{T}} u(T_{\infty} - T) dy = \alpha \left(\frac{\partial T}{\partial y}\right)_{y=0}$$

Information for part (c): When we use the parabolic temperature profile  $(T_w - T)/(T_w - T_\infty) = (p/2)(3-p^2)$ , where  $p = y/\delta_T$ ; and when the velocity boundary layer thickness is given by  $\delta(x) = 4.64x/\sqrt{(Re_x)}$  with  $Re_x = U_\infty x/v$ ; then the integral analysis using the above equation produces the following expression for the local Nusselt number:

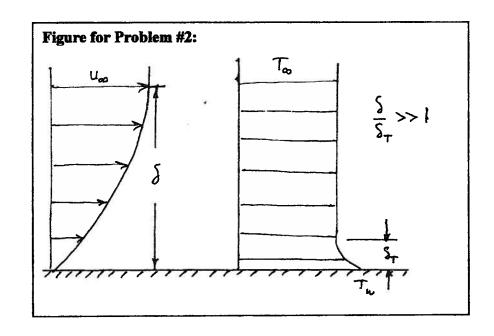
$$Nu_x = CPr^{1/3}Re_x^{1/2}$$

$$C = 0.331,$$

$$Re_x = U_{\infty}x/v$$

$$Pr = v/\alpha.$$

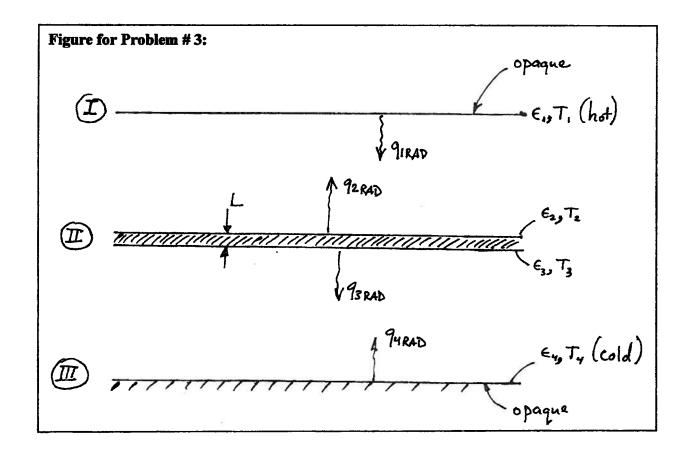
c) For a plate of length L = 3m calculate the difference between the heat transfer over 0 < x < 3m and 1.5m < x < 3m. Find the distance, expressed as a fraction of L, at which the heat transfer to both sections is identical.



#### Problem 3:

Consider the steady state heat transfer problem shown in the figure below. For part (a) of this question heat transfer occurs only by radiation. In part (b) conduction is allowed.

- Hot surface 1 emits flux  $q_{RADI}$  at  $T_I = 1650K$ ,  $\varepsilon_I = 0.05$ . Layer I (which can be thought of as a flame, is *optically transparent*).
- Surface 2 emits flux  $q_{RAD2}$  at  $T_2$ ,  $\varepsilon_2 = 0.95$  toward the top of the region.
- Surface 3 emits flux  $q_{RAD3}$  at  $T_3$ ,  $\varepsilon_3 = 0.95$  toward surface 4.
- Surface 4 emits flux  $q_{RAD4}$  at  $T_4 = 300K$ ,  $\varepsilon_4 = 0.5$ . The material III is optically transparent.
- Medium II is optically opaque.
- a) If surfaces 2, 3 of medium II coincide (i.e., if  $L \to 0$  so that  $T_2 = T_3$ ) solve for the temperature of medium II.
- b) If medium II has thickness  $L \neq 0$ , set up the radiation calculation and calculate the temperatures  $T_2$  and  $T_3$ . HINT: The conductive heat flux through medium II in the steady state is given by  $q'' = -k(T_3 T_2)/L$  where k is the conductivity of medium II and L is its thickness. Use k = 1 W/mK, L = 0.3 mm,  $q'' = 3 \times 10^4$  W/m<sup>2</sup>.



# **Question #4**

A solar collector is made of an absorber surface with emissivity of 0.15 and absorptivity of 0.9. The collector has no cover plate. At a given time of the day, the absorber surface temperature  $T_s$  is 15°C when the solar irradiation is 900 W/m², the effective sky temperature is -6°C and the ambient air temperature is 35°C. Assume that the heat transfer convection coefficient for a calm day is estimated as:  $h=0.22(T_s-T_0)^{1/3}$  W/m².K

- (a) Calculate the net heat removal from the collector for the above condition.
- (b) Calculate the efficiency of the collector.
- (c) To improve collector's efficiency a cover plat is often used. Why? Can you explain by numbers.