

**Department of Mechanical Engineering
Michigan State University
East Lansing, Michigan**

**Ph.D. Qualifying Exam in
Fluid Mechanics**

- Closed book, but one sheet (8.5" x 11", front and back) of your own notes with equations permitted.
- Some basic equations are provided on an attached information sheet.
- Answer all questions.
- All questions have the same weighting.

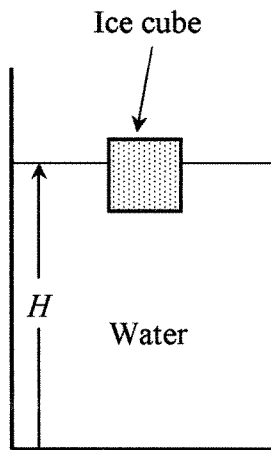
Exam prepared by

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Problem 1:

An ice cube floats on the water contained in a can. The initial height of the water is H . After a while the ice melts. What happens to the height of the water? Does it go up, go down, or stay the same? Explain in details how you arrive at your answer. Assume water evaporation is negligible during the time that it takes for the ice to melt.

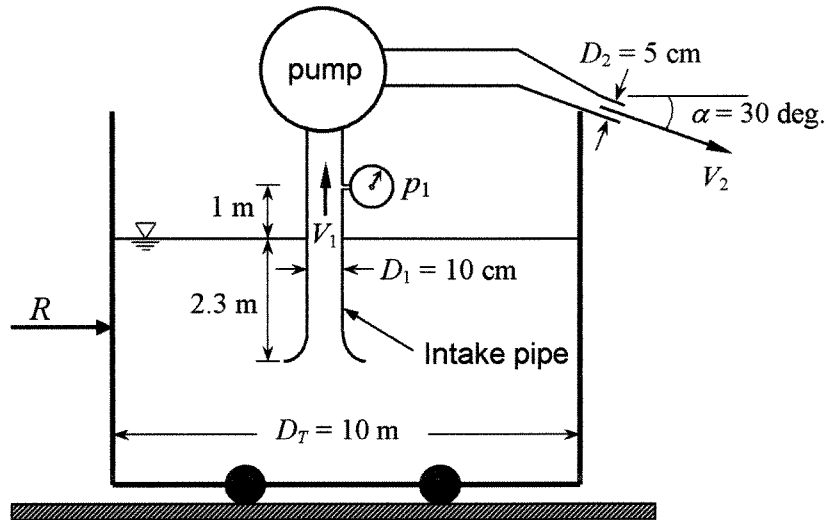


Problem 2:

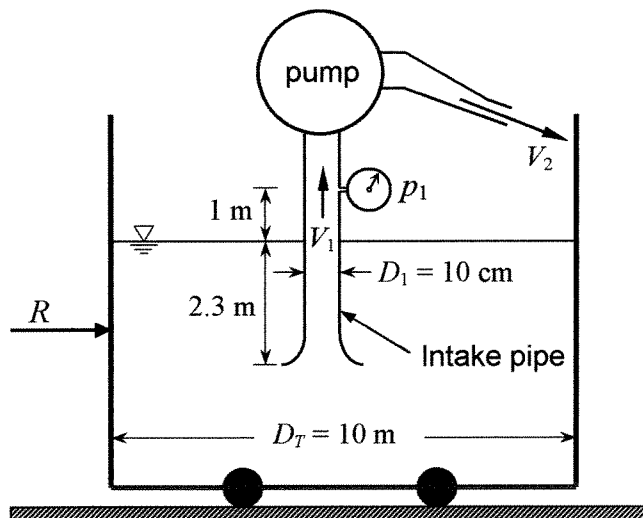
A pump takes in water from a large tank and discharges it into ambient outside the tank with a speed V_2 at an angle $\alpha = 30$ degrees relative to horizontal; see schematic below. The flow can be assumed steady. The tank is held in a stationary position by a force R . The flow during the intake up to the pump, and discharge after the pump can be assumed frictionless and without losses. The flow through the pump is not frictionless. A pressure tap located in the intake pipe, at the position indicated in the sketch, measures a gage pressure of $(p_1)_{\text{gage}} = -22.3$ kPa.

(a) Determine the water speed V_1 (in m/s) in the intake pipe, the water speed at the nozzle exit V_2 (in m/s) and the mass flow rate \dot{m} (in kg/s) going through this flow system.

(b) Determine the expression for the force R in terms of the variable given in the problem, and then compute the numerical value of R . Assume the friction in the wheels is negligible.

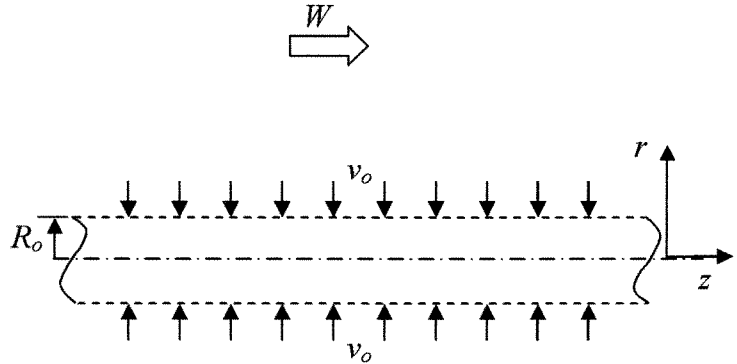


(c) Suppose the length of the outlet pipe from the pipe is now reduced so that the water from the nozzle exit discharges into the tank as shown in the sketch below. All other flow parameters remain the same as before. Determine the force R under these conditions.



Problem 3:

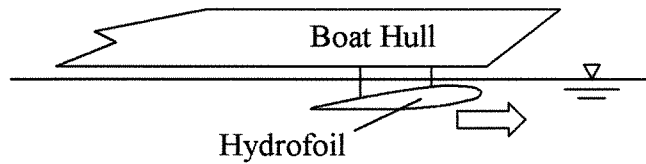
Consider the steady incompressible flow along an infinitely long circular cylinder of radius R_o as shown in the figure. If a uniform radial suction velocity of v_o is applied at the surface of the cylinder, and the axial flow velocity far away from the cylinder is W :



- Find the velocity field around the cylinder. Hint: you may assume the axial velocity component to be of the form: $v_z = Cr^n$; where C and n are constants, and r is the radial coordinate in a cylindrical coordinate system. You may also neglect any variations in the velocity and pressure field in the z direction. State and justify any other physically reasonable assumptions if necessary.
- Determine the 99% boundary layer thickness relative to R_o ; i.e., $\delta_{.99}/R_o$, for a Reynolds number based on the R_o and W of 10. Compare the result to that of a Reynolds number of 1000.

Problem 4:

An engineer is designing a hydrofoil for a high-speed sailboat. At high boat velocities, the hydrofoil, which is attached to the bottom of the boat (see figure), will lift the boat outside the water to minimize drag forces and achieve a design speed of 20 m/s.



The engineer selects a NACA 0012 airfoil shape for the hydrofoil to be mounted at a nominal angle of attack of 8° . The chord length of the hydrofoil is 1 m and its span is 2 m. To determine the forces expected to act on the hydrofoil, the engineer plans to test a 1:4 scale model of the hydrofoil.

- Does it matter if the test is conducted in air or water?
- Is it possible to obtain *full* dynamic similarity in the test? If no, how would this affect the validity of the test? (You may find the data given below of the lift and drag coefficients on NACA 0012 at different Reynolds numbers helpful in addressing this question)
- What flow velocity should the engineer employ for the test? If the measured lift and drag forces acting on the hydrofoil are 40 and 6 kN, respectively, what should the corresponding forces acting on the full-scale hydrofoil be? (You may take the kinematic viscosity and density of water to be $1.14 \times 10^{-6} \text{ m}^2/\text{s}$ and 1000 kg/m^3 , respectively. Corresponding values for air are $14.5 \times 10^{-6} \text{ m}^2/\text{s}$ and 1.2 kg/m^3)

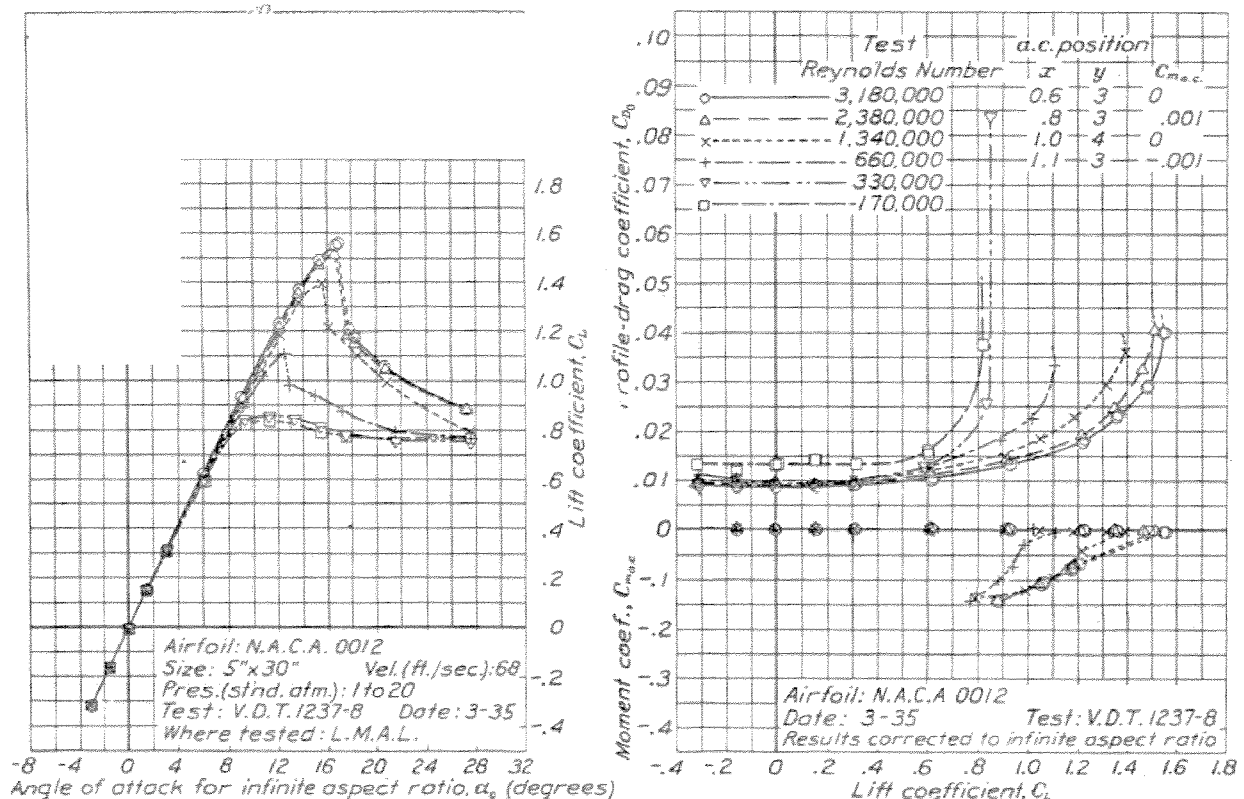


FIGURE 3.—N. A. C. A. 0012.

Integral mass conservation equation:

$$0 = \frac{\partial}{\partial t} \int_{CV} \rho dV + \int_{CS} \rho \vec{V} \cdot d\vec{A}$$

Integral momentum equation:

$$\vec{F} = \vec{F}_s + \vec{F}_B = \frac{\partial}{\partial t} \int_{CV} \vec{V} \rho dV + \int_{CS} \vec{V} \rho \vec{V} \cdot d\vec{A}$$

Table C-1 The Equation of Continuity

Rectangular Coordinates (x, y, z):

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

Cylindrical Coordinates (r, θ, z):

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r} \frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

Spherical Coordinates (r, θ, φ):

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r}(\rho r^2 v_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta}(\rho v_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \varphi}(\rho v_\varphi) = 0$$

Table C-5 Momentum Equations for a Newtonian Fluid with Constant Density (ρ) and Constant Viscosity (μ)

Rectangular Coordinates (x, y, z):

$$\rho \left(\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} \right) = \mu \left[\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2} \right] - \frac{\partial p}{\partial x} + \rho g_x$$

$$\rho \left(\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z} \right) = \mu \left[\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2} \right] - \frac{\partial p}{\partial y} + \rho g_y$$

$$\rho \left(\frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} \right) = \mu \left[\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2} \right] - \frac{\partial p}{\partial z} + \rho g_z$$

Cylindrical Coordinates (r, θ, z):

$$\rho \left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z} \right) = \mu \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} (r v_r) \right) + \frac{1}{r^2} \frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2} \frac{\partial v_\theta}{\partial \theta} \right] - \frac{\partial p}{\partial r} + \rho g_r$$

$$\rho \left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z \frac{\partial v_\theta}{\partial z} \right) = \mu \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} (r v_\theta) \right) + \frac{1}{r^2} \frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} \right] - \frac{1}{r} \frac{\partial p}{\partial \theta} + \rho g_\theta$$

$$\rho \left(\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z} \right) = \mu \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2} \right] - \frac{\partial p}{\partial z} + \rho g_z$$

Table C-3 Components of the Stress Tensor for Newtonian Fluids

Rectangular Coordinates (x, y, z)	Cylindrical Coordinates (r, θ, z)
$\tau_{xx} = \mu \left[2 \frac{\partial v_x}{\partial x} - \frac{2}{3} (\nabla \cdot \mathbf{v}) \right]$	$\tau_{rr} = \mu \left[2 \frac{\partial v_r}{\partial r} - \frac{2}{3} (\nabla \cdot \mathbf{v}) \right]$
$\tau_{yy} = \mu \left[2 \frac{\partial v_y}{\partial y} - \frac{2}{3} (\nabla \cdot \mathbf{v}) \right]$	$\tau_{\theta\theta} = \mu \left[2 \left(\frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} \right) - \frac{2}{3} (\nabla \cdot \mathbf{v}) \right]$
$\tau_{zz} = \mu \left[2 \frac{\partial v_z}{\partial z} - \frac{2}{3} (\nabla \cdot \mathbf{v}) \right]$	$\tau_{zz} = \mu \left[2 \frac{\partial v_z}{\partial z} - \frac{2}{3} (\nabla \cdot \mathbf{v}) \right]$
$\tau_{xy} = \tau_{yx} = \mu \left[\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right]$	$\tau_{r\theta} = \tau_{\theta r} = \mu \left[r \frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right) + \frac{1}{r} \frac{\partial v_r}{\partial \theta} \right]$
$\tau_{yz} = \tau_{zy} = \mu \left[\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y} \right]$	$\tau_{\theta z} = \tau_{z\theta} = \mu \left[\frac{\partial v_\theta}{\partial z} + \frac{1}{r} \frac{\partial v_z}{\partial \theta} \right]$
$\tau_{zx} = \tau_{xz} = \mu \left[\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z} \right]$	$\tau_{zr} = \tau_{rz} = \mu \left[\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z} \right]$
$(\nabla \cdot \mathbf{v}) = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$	$(\nabla \cdot \mathbf{v}) = \frac{1}{r} \frac{\partial}{\partial r} (rv_r) + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z}$