Student ID ____________________

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Ph.D. Qualifying Exam in Fluid Mechanics

- Closed book, but one sheet (8.5" x 11", front and back) of your own notes with equations permitted.
- Some basic equations are provided on an attached information sheet.
- Answer all questions.
- All questions have the same weighting.

Exam prepared by

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Problem 1:

The propeller of a 150 m long ocean-going cargo ship is 2.5 m in diameter and rotates at 100 rpm. In such ships, the propeller is typically positioned a small distance below the ocean surface. If a 1:100 scale model of the ship and propeller is to be tested in a water tank, and effects of water viscosity can be neglected,

1. Find the dimensionless groups that characterize the drag force on the ship that the thrust of its propeller must balance, and give their names.

2. At what rpm should the model's propeller rotate?

3. If the drag coefficients of the real and model ships are the same, what is the ratio between the power requirements of the model and the full-size ship?

Problem 2:

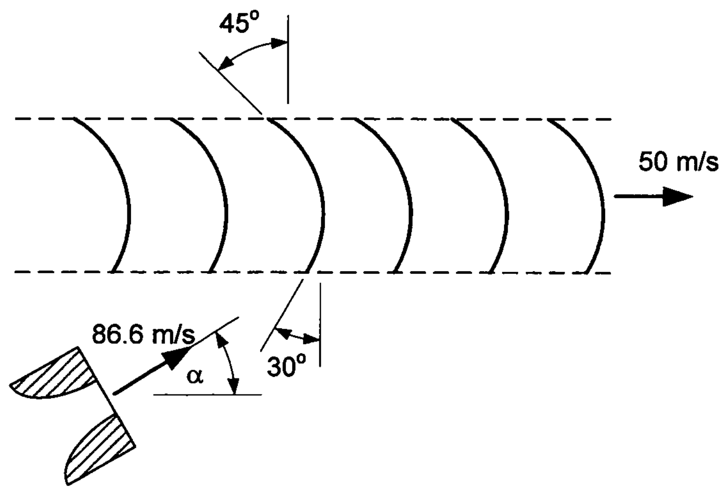
A viscous Newtonian fluid flows down a vertical tube of diameter D under the influence of gravity. The flow is assumed to be fully developed along its entire length and both ends of the tube are at atmospheric pressure. Find an expression for the mean velocity U in terms of g, D and other parameters of the problem.

Problem 3:

Consider a series of turning vanes struck by a continuous jet of water (density = 1000 kg/m^3), as shown in the figure below) that leaves a 50 mm diameter nozzle at constant speed of 86.6 m/s. The vanes move with a constant speed of 50 m/s. Note that all the mass flow leaving the jet crosses the vanes. The curvature of the vanes is described by the angles shown in the figure below. Ignoring friction effects on the flow over the vane:

a) Evaluate the nozzle angle, α, required to assure that the jet enters tangent to the leading edge of each vane.

b) Calculate the force that must be applied to maintain the vane speed constant.



Problem 4:

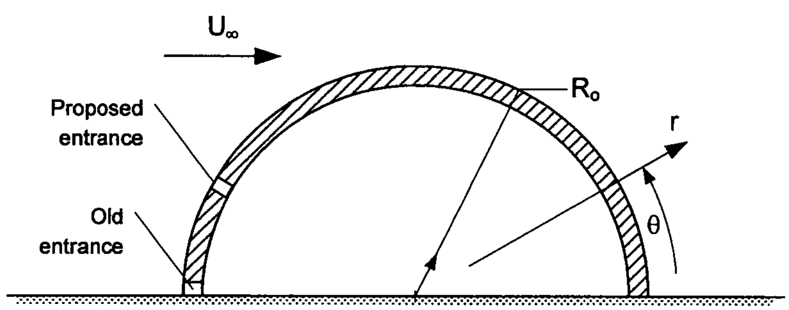
An arctic hut in the shape of a half-circular cylinder is of radius R_o. A wind of velocity U_∞ batters the hut and threatens to raise it off its foundations due to the lift of the wind. This lift is partly due to the fact that the entrance to the hut is at the ground level at the location of the stagnation pressure. It is proposed to raise the entrance from the ground level (as shown in the figure) to a position at which the net lift force on the hut would vanish. To determine the angle θ at which the entrance should be positioned, the flow is assumed to be incompressible, inviscid and irrotational. Under these assumptions, the two-dimensional wind velocity field around the hut is given by:

$$v_r = U_\infty\left(1 - \frac{R_o^2}{r^2}\right)\cos\theta$$

$$v_\theta = -U_\infty\left(1 + \frac{R_o^2}{r^2}\right)\sin\theta$$

where r and θ are cylindrical coordinates as defined in the figure below. Using the above information find the angle θ of the new entrance that will nullify the net lift force on the hut.

(Hint: $\int \sin^3(ax)dx = -\frac{\cos(ax)}{a} + \frac{\cos^3(ax)}{3a}$)



Problem 5:

1. Two soap bubbles, A and B, are of identical composition and are connected by a thin tube. The bubbles are initially of equal size. Bubble A is then perturbed so that its diameter momentarily exceeds that of bubble B. What happens next, and why?

2. When driven from the tee, a dimpled golf ball travels faster than a smooth one of equal diameter. Explain why. Also, explain whether a dimpled tennis ball would travel faster than a smooth one when served.

3. If the height of a levee is increased by 20%, to provide better protection against higher water levels caused by hurricane activity, how much greater a hydrostatic force must it also be able to withstand?

4. An aircraft takes off from a runway at a fixed speed on two different days—a cold day and a warm day. On which day would it reach its cruising altitude sooner, and why?

Integral mass conservation equation:

$$0=\frac{\partial}{\partial t}\int_{CV}\rho d\forall+\int_{CS}\rho\vec{V}.d\vec{A}$$

Integral momentum equation:

$$\vec{F}=\vec{F}_s+\vec{F}_B=\frac{\partial}{\partial t}\int_{CV}\vec{V}\rho d\forall+\int_{CS}\vec{V}\rho\vec{V}.d\vec{A}$$

Integral angular momentum equation:

$$\sum(\vec{r}\times\vec{F})=\frac{\partial}{\partial t}\int_{CV}(\vec{r}\times\vec{V})\rho d\forall+\int_{CS}(\vec{r}\times\vec{V})\rho\vec{V}\cdot d\vec{A}$$

The Equation of Continuity

Rectangular Coordinates (x, y, z):

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x}(\rho v_x)+\frac{\partial}{\partial y}(\rho v_y)+\frac{\partial}{\partial z}(\rho v_z)=0$$

Cylindrical Coordinates (r, θ, z):

$$\frac{\partial\rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r)+\frac{1}{r}\frac{\partial}{\partial\theta}(\rho v_\theta)+\frac{\partial}{\partial z}(\rho v_z)=0$$

Momentum Equations for Incompressible Newtonian Fluid with Constant Viscosity (μ)

Rectangular Coordinates (x, y, z):

$$\rho\left(\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}\right)=\mu\left[\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right]-\frac{\partial p}{\partial x}+\rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t}+v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}+v_z\frac{\partial v_y}{\partial z}\right)=\mu\left[\frac{\partial^2 v_y}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}+\frac{\partial^2 v_y}{\partial z^2}\right]-\frac{\partial p}{\partial y}+\rho g_y$$

$$\rho\left(\frac{\partial v_z}{\partial t}+v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z}\right)=\mu\left[\frac{\partial^2 v_z}{\partial x^2}+\frac{\partial^2 v_z}{\partial y^2}+\frac{\partial^2 v_z}{\partial z^2}\right]-\frac{\partial p}{\partial z}+\rho g_z$$

Cylindrical Coordinates (r, θ, z):

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial\theta}-\frac{v_\theta^2}{r}+v_z\frac{\partial v_r}{\partial z}\right)=\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right)+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial\theta^2}+\frac{\partial^2 v_r}{\partial z^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial\theta}\right]-\frac{\partial p}{\partial r}+\rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial\theta}+\frac{v_r v_\theta}{r}+v_z\frac{\partial v_\theta}{\partial z}\right)=\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right)+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial\theta^2}+\frac{\partial^2 v_\theta}{\partial z^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial\theta}\right]-\frac{1}{r}\frac{\partial p}{\partial\theta}+\rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial\theta}+v_z\frac{\partial v_z}{\partial z}\right)=\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial\theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right]-\frac{\partial p}{\partial z}+\rho g_z$$

Table C-3 Components of the Stress Tensor for Newtonian Fluids

Rectangular Coordinates (x, y, z)	Cylindrical Coordinates (r, θ, z)
$\tau_{xx} = \mu\left[2\frac{\partial v_x}{\partial x} - \frac{2}{3}(\nabla \cdot \mathbf{v})\right]$	$\tau_{rr} = \mu\left[2\frac{\partial v_r}{\partial r} - \frac{2}{3}(\nabla \cdot \mathbf{v})\right]$
$\tau_{yy} = \mu\left[2\frac{\partial v_y}{\partial y} - \frac{2}{3}(\nabla \cdot \mathbf{v})\right]$	$\tau_{\theta\theta} = \mu\left[2\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right) - \frac{2}{3}(\nabla \cdot \mathbf{v})\right]$
$\tau_{zz} = \mu\left[2\frac{\partial v_z}{\partial z} - \frac{2}{3}(\nabla \cdot \mathbf{v})\right]$	$\tau_{zz} = \mu\left[2\frac{\partial v_z}{\partial z} - \frac{2}{3}(\nabla \cdot \mathbf{v})\right]$
$\tau_{xy} = \tau_{yx} = \mu\left[\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right]$	$\tau_{r\theta} = \tau_{\theta r} = \mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]$
$\tau_{yz} = \tau_{zy} = \mu\left[\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right]$	$\tau_{\theta z} = \tau_{z\theta} = \mu\left[\frac{\partial v_\theta}{\partial z} + \frac{1}{r}\frac{\partial v_z}{\partial \theta}\right]$
$\tau_{zx} = \tau_{xz} = \mu\left[\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z}\right]$	$\tau_{zr} = \tau_{rz} = \mu\left[\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right]$
$(\nabla \cdot \mathbf{v}) = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$	$(\nabla \cdot \mathbf{v}) = \frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z}$

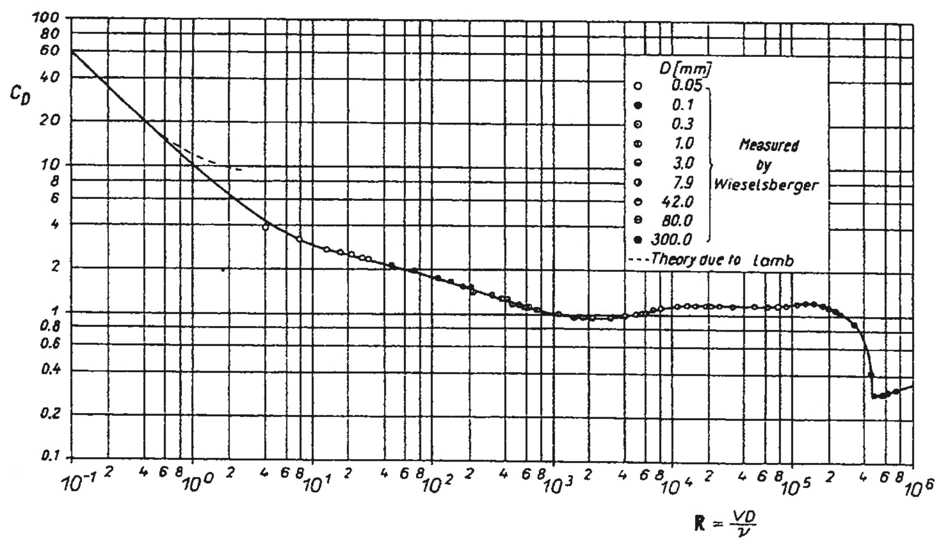
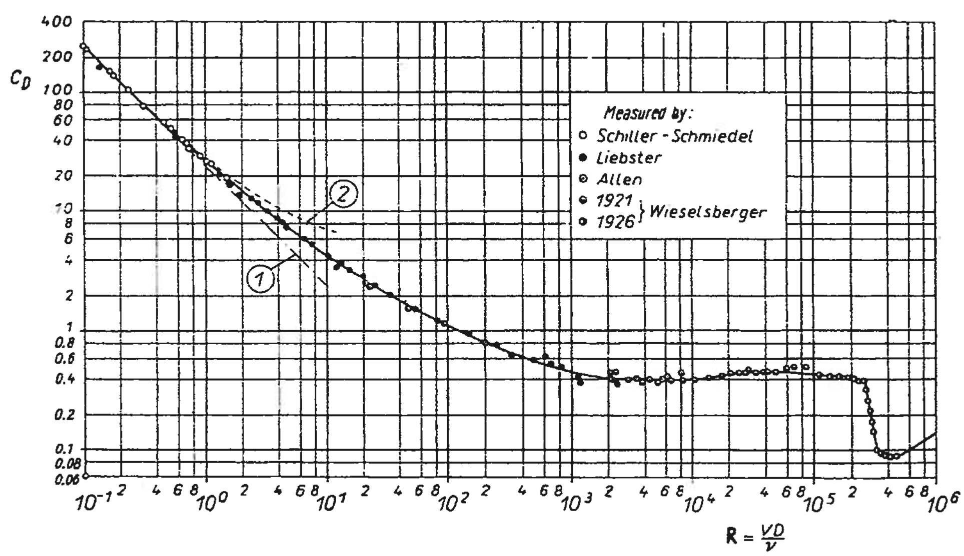


Fig. 1.4. Drag coefficient for circular cylinders as a function of the Reynolds number



Curve (1), Stokes' theory: $C_D = 24/\mathbf{R}$

Fig. 1.5. Drag coefficient for spheres as a function of the Reynolds number
Curve (1): Stokes's theory, eqn. (6.10); curve (2): Oseen's theory, eqn. (6.13)