Student ID ____________________

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Ph.D. Qualifying Exam in Fluid Mechanics

- Closed book, but one sheet (8.5" x 11", front and back) of your own notes with equations permitted.
- Some basic equations are provided on an attached information sheet.
- Answer all questions.
- All questions have the same weighting.

Exam prepared by

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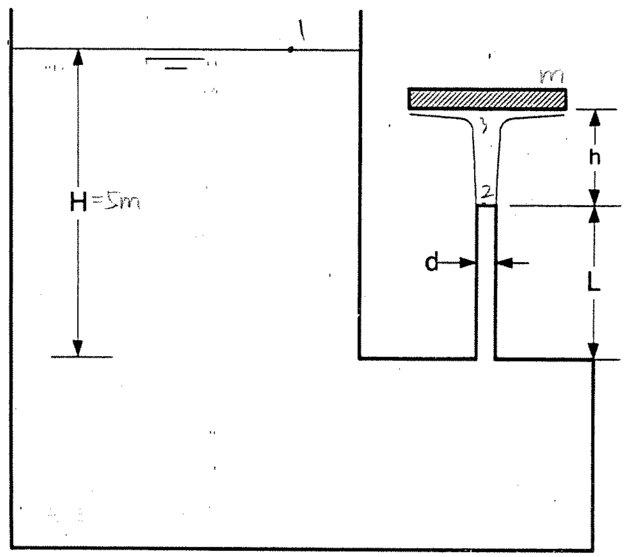
Problem 1:

A water (density = 1000 kg/m^3) jet issuing from a tank is used to support a disc with mass *m* as shown in the figure below. Assuming H = 5 m, d = 25 mm and *M* = 1 kg, do the following:

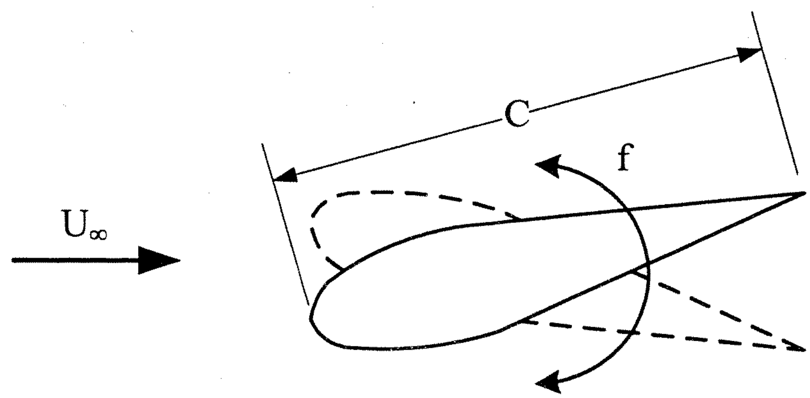


(a) Ignoring viscous losses, determine the height h of the disc as function of the pipe length L. Calculate the largest possible value of L and corresponding value of h in meters. Also, obtain the largest possible value of h and the corresponding L value in meters?

(b) If the losses in the system are characterized by a loss coefficient K = 0.3 (based on the dynamic head at the exit of the pipe) when L = 0.25 m, what is the value of h? What is the corresponding value when the losses are neglected?

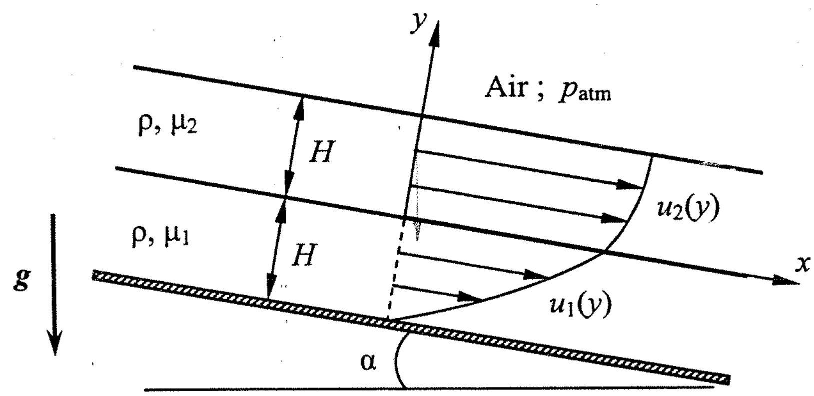
Problem 2:

Consider the two-dimensional airflow over an oscillating airfoil, as shown in the figure below. The airfoil has a chord length C and oscillates at a frequency of f. The approaching flow freestream velocity is U_∞. Reduce the differential form of the governing equations associated with this problem into a non-dimensional form. Based on this, identify the relevant non-dimensional parameters. If it is planned to test the airfoil in water by employing a model of one-tenth the actual airfoil size, determine the appropriate values of C, f and U_∞ for the test if the corresponding actual-airfoil parameters are: 1m, 10 Hz, and 10 m/s, respectively. You may assume the kinematic viscosity of water to be one-tenth that of air. $\frac{1}{10}$



Problem 3:

Two superposed layers of immiscible liquid (for exampe, oil over water) of equal density ρ, each of constant uniform thickness H, flow due to gravity down an inclined plane at angle α. The lower liquid has a coefficient of viscosity μ_1. The upper liquid is much more viscous and has a higher coefficient of viscosity μ_2. The flow is two-dimensional, steady, and parallel to the plane. The free surface is exposed to constant atmospheric pressure, with negligible shear stress on the free surface. Use the coordinate system indicated below.



(a) Reduce the equations of motion for this problem and derive the differential equation satisfied by the velocity profiles $u_1(y)$ and $u_2(y)$ in the two fluids. You must explain all your steps, i.e. what are the consequences of the continuity equation, what happens to various terms in the Navier-Stokes equations, and what happens to the pressure field. Clearly indicate the boundary conditions for this problem.

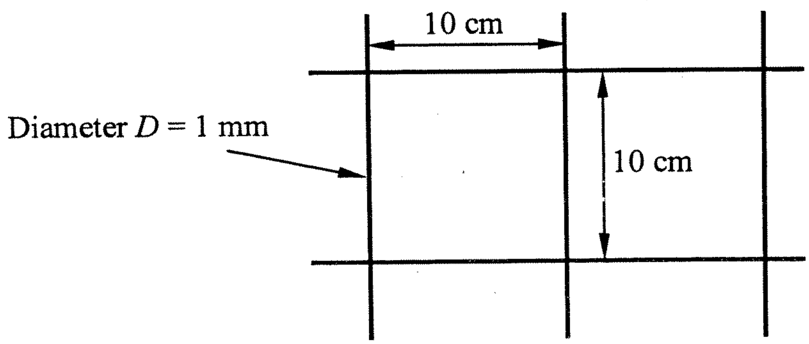
(b) Derive the shape of the velocity profiles $u_1(y)$ and $u_2(y)$ in the two liquid layers.

(c) Derive how much the velocity changes across the top, more viscous, layer in terms of the velocity ratio across the layer $\dfrac{u_2(y=H)}{u_2(y=0)}$ as a function of the viscosity ratio $M \equiv \mu_2/\mu_1$. Show that for a very viscous oil layer over water with $M = 100$ the velocity in the oil layers hardly varies at all.

(d) **Problem 4:**

A fishnet is made of 1-mm diameter nylon strings tied into squares 10 cm per side. A fishing boat is towing is seawater a 10 m × 10 m section of this net normal to its plane at a speed of 1/8 m/s. For seawater, $\rho = 1030\ kg/m^3$, and $\mu = 0.0012\ kg/(m.s)$.

Estimate the force and the horsepower required to tow the fishnet.



A piece of the fishnet

Integral mass conservation equation:

$$0=\frac{\partial}{\partial t}\int_{CV}\rho\, d\forall+\int_{CS}\rho\vec{V}.d\vec{A}$$

Integral momentum equation:

$$\vec{F}=\vec{F}_s+\vec{F}_B=\frac{\partial}{\partial t}\int_{CV}\vec{V}\rho\, d\forall+\int_{CS}\vec{V}\rho\vec{V}.d\vec{A}$$

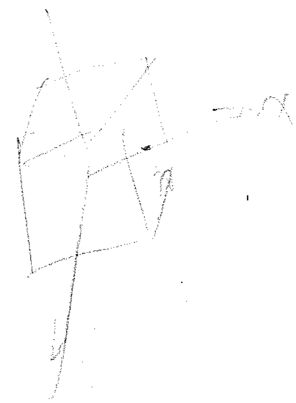


Table C-1 The Equation of Continuity

Rectangular Coordinates (x, y, z):

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x}(\rho v_x)+\frac{\partial}{\partial y}(\rho v_y)+\frac{\partial}{\partial z}(\rho v_z)=0$$

Cylindrical Coordinates (r, θ, z):

$$\frac{\partial\rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r)+\frac{1}{r}\frac{\partial}{\partial\theta}(\rho v_\theta)+\frac{\partial}{\partial z}(\rho v_z)=0$$

Spherical Coordinates (r, θ, φ):

$$\frac{\partial\rho}{\partial t}+\frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 v_r)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\rho v_\theta\sin\theta)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}(\rho v_\varphi)=0$$

Table C-5 Momentum Equations for a Newtonian Fluid with Constant Density (ρ) and Constant Viscosity (μ)

Rectangular Coordinates (x, y, z):

$$\rho\left(\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}\right)=\mu\left[\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right]-\frac{\partial p}{\partial x}+\rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t}+v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}+v_z\frac{\partial v_y}{\partial z}\right)=\mu\left[\frac{\partial^2 v_y}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}+\frac{\partial^2 v_y}{\partial z^2}\right]-\frac{\partial p}{\partial y}+\rho g_y$$

$$\rho\left(\frac{\partial v_z}{\partial t}+v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z}\right)=\mu\left[\frac{\partial^2 v_z}{\partial x^2}+\frac{\partial^2 v_z}{\partial y^2}+\frac{\partial^2 v_z}{\partial z^2}\right]-\frac{\partial p}{\partial z}+\rho g_z$$

Cylindrical Coordinates (r, θ, z):

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial\theta}-\frac{v_\theta^2}{r}+v_z\frac{\partial v_r}{\partial z}\right)=\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right)+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial\theta^2}+\frac{\partial^2 v_r}{\partial z^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial\theta}\right]-\frac{\partial p}{\partial r}+\rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial\theta}+\frac{v_r v_\theta}{r}+v_z\frac{\partial v_\theta}{\partial z}\right)=\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right)+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial\theta^2}+\frac{\partial^2 v_\theta}{\partial z^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial\theta}\right]-\frac{1}{r}\frac{\partial p}{\partial\theta}+\rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial\theta}+v_z\frac{\partial v_z}{\partial z}\right)=\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial\theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right]-\frac{\partial p}{\partial z}+\rho g_z$$

Table C-3 Components of the Stress Tensor for Newtonian Fluids

Rectangular Coordinates (x, y, z)	Cylindrical Coordinates (r, θ, z)
$\tau_{xx} = \mu\left[2\frac{\partial v_x}{\partial x} - \frac{2}{3}(\nabla \cdot \mathbf{v})\right]$	$\tau_{rr} = \mu\left[2\frac{\partial v_r}{\partial r} - \frac{2}{3}(\nabla \cdot \mathbf{v})\right]$
$\tau_{yy} = \mu\left[2\frac{\partial v_y}{\partial y} - \frac{2}{3}(\nabla \cdot \mathbf{v})\right]$	$\tau_{\theta\theta} = \mu\left[2\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right) - \frac{2}{3}(\nabla \cdot \mathbf{v})\right]$
$\tau_{zz} = \mu\left[2\frac{\partial v_z}{\partial z} - \frac{2}{3}(\nabla \cdot \mathbf{v})\right]$	$\tau_{zz} = \mu\left[2\frac{\partial v_z}{\partial z} - \frac{2}{3}(\nabla \cdot \mathbf{v})\right]$
$\tau_{xy} = \tau_{yx} = \mu\left[\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right]$	$\tau_{r\theta} = \tau_{\theta r} = \mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]$
$\tau_{yz} = \tau_{zy} = \mu\left[\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right]$	$\tau_{\theta z} = \tau_{z\theta} = \mu\left[\frac{\partial v_\theta}{\partial z} + \frac{1}{r}\frac{\partial v_z}{\partial \theta}\right]$
$\tau_{zx} = \tau_{xz} = \mu\left[\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z}\right]$	$\tau_{zr} = \tau_{rz} = \mu\left[\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right]$
$(\nabla \cdot \mathbf{v}) = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$	$(\nabla \cdot \mathbf{v}) = \frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z}$

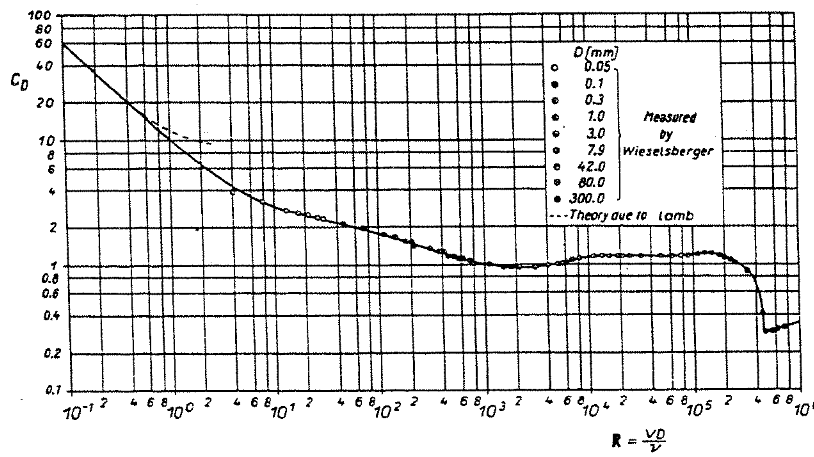


Fig. 1.4. Drag coefficient for circular cylinders as a function of the Reynolds number

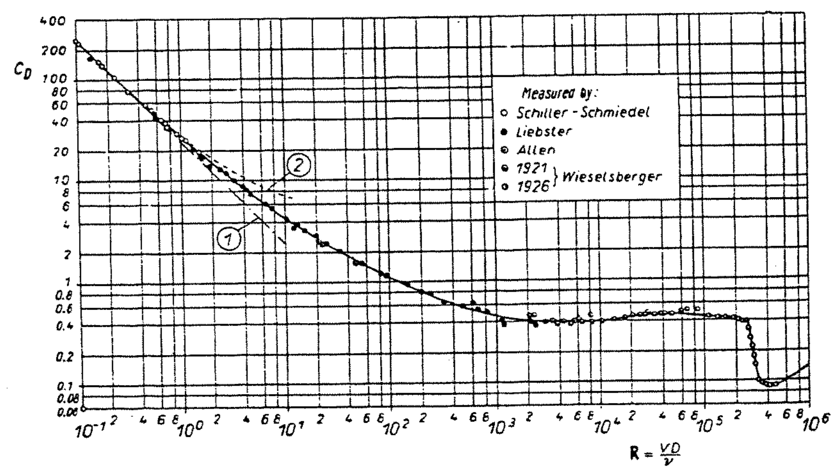


Fig. 1.5. Drag coefficient for spheres as a function of the Reynolds number
Curve (1): Stokes's theory, eqn. (6.10); curve (2): Oseen's theory, eqn. (6.13)