Code No.________________________

Ph.D. Qualifier Exam — Fluid Mechanics

Department of Mechanical Engineering
Michigan State University

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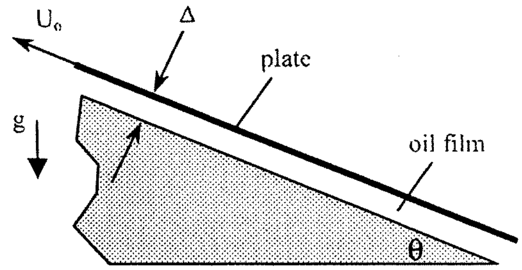
Directions: Closed book

All problems carry equal weight.

Exam prepared by Profs. Brereton and Naguib

Problem 1

An infinitely long and wide plate is pulled up an infinitely long and wide ramp with incliniation angle θ, as shown in the figure. The plate is made to move with velocity U_o over a thin oil layer to reduce friction between the plate and the ramp. Do the following:



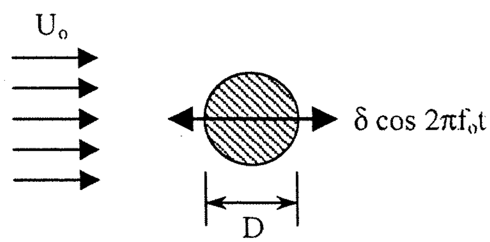
a) Obtain an equation describing the velocity field within the oil film (state and justify all assumptions clearly)

b) Is there an optimal value of U_o for which the force exerted by the oil film on the ramp is minimum? If so, what is the value of this velocity in terms of parameters given in the figure and fluid properties? What is the corresponding minimum force value?

c) Is the net flow rate of the oil up or down the ramp? Provide evidence for your answer

Problem 2

It is desired to desired to build a micro device in which a 10 μm-diameter cylinder oscilates at a frequency of 100 kHz with/against an air flow of uniform velocity (U_o) of 10 m/s. The amplitude of oscillation (δ) is anticipated to be 1 μm. For the purposes of designing the device, it is desired to estimate the unsteady flow force acting on the cylinder (which may be assumed infinitely long).



Because of the difficulty and cost in fabrication of the micro device as well as the difficulty in measuring forces acting on such a small object, an engineer proposes to conduct a scale-up test in which the cylinder diameter is 5 mm. What should be the values for U_o, δ and f_o for the test? Should water or air be used for the test? What is the relation between the measured and actual force acting on the device?

Problem 3.

Water flows through a structure that comprises a reducing elbow with an oil pipe that passes through it and is welded to it. If the flows of water and oil through this structure are steady and the elbow is stationary, find:

(*i*) the horizontal component of force exerted by the water on the elbow/pipe structure;

(*ii*) the horizontal component of force exerted by the surrounding air on the elbow;

(*iii*) the horizontal component of force exerted by the oil on the elbow/pipe structure;

(*iv*) the total horizontal component of force exerted by the water, air and oil on the elbow/pipe structure.

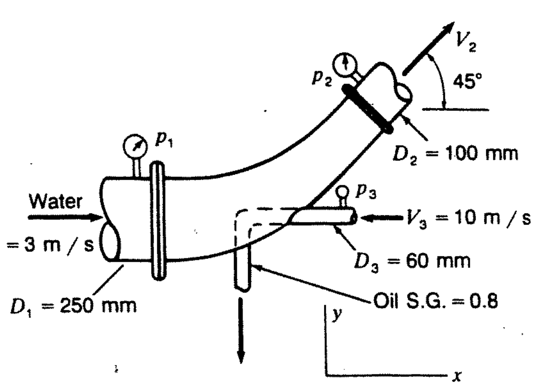
Identify clearly any control volumes you might use in any part of the problem.

Problem 4.

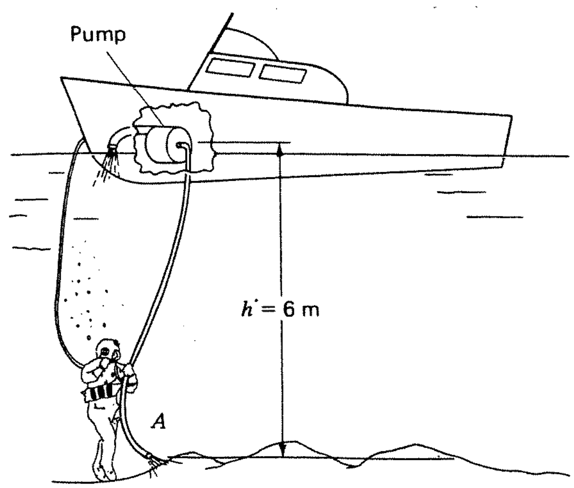
A diver directs a flexible pipe into which sand and water are sucked by a 2-kW pump, to expose wreckage on the sea bed. If the pressure at inlet A is approximately the hydrostatic pressure of the surrounding water, the specific gravity of the sand/water mixture is 1.8, and the pipe diameter is 250 mm, so frictional losses can be ignored, how much sand will be sucked up per second?

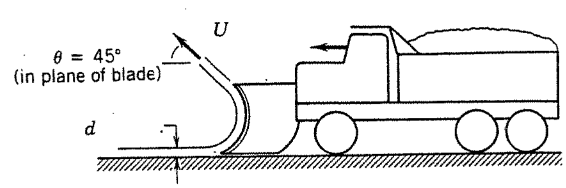
Problem 5.

A snow plough mounted on a truck clears a path through wet, heavy snow. The plow blade is 4 m wide, the snow is $d = 0.25$ m deep, and its density is 150 kg/m^3. Snow is discharged at 45° from the horizontal *and* at 45° from the travel direction. What force is required to push the plough when the mass of the truck is 8,000 kg and it moves at $U = 8$ m/s?



Data: $p_1 = 250$ kPa gage
$p_2 = 180$ kPa gage
$p_3 = 200$ kPa gage
$p_{atm} = 101.325$ kPa





Additional Information

$\nu_{air} = 1.5 \times 10^{-5}\ m^2/s$
$\nu_{water} = 1.14 \times 10^{-6}\ m^2/s$

For an incompressible, Newtonian fluid, the governing and constitutive equations are:

Mass Conservation Equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Momentum Equation

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \rho g_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = \rho g_y - \frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = \rho g_z - \frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$

Stress Tensor Components

$$\tau_{xy} = \tau_{yx} = \mu\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)$$

$$\tau_{yz} = \tau_{zy} = \mu\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)$$

$$\tau_{zx} = \tau_{xz} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)$$

$$\sigma_{xx} = -p - \frac{2}{3}\mu\nabla\cdot\vec{V} + 2\mu\frac{\partial u}{\partial x}$$

$$\sigma_{yy} = -p - \frac{2}{3}\mu\nabla\cdot\vec{V} + 2\mu\frac{\partial v}{\partial y}$$

$$\sigma_{zz} = -p - \frac{2}{3}\mu\nabla\cdot\vec{V} + 2\mu\frac{\partial w}{\partial z}$$