Ph.D. Qualifying Exam in Fluid Mechanics

• Closed book, but one sheet (8.5” x 11”, front and back) of your own notes with equations permitted.
• Some basic equations are provided on an attached information sheet.
• Answer all questions.
• All questions have the same weighting.

Exam prepared by

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**Problem 1:**

The radiator fan on an automobile engine is a source of considerable noise. The noise power $P$ produced by the fan depends on its diameter $D$ and its angular velocity $\omega$, as well as on the air density $\rho$ and the speed of sound in air $c$.

1. Find the dimensionless variables that characterize the noise power $P$ in this application, if $\rho$, $D$ and $\omega$ are used as repeating variables.

2. If a real fan rotates at 1000 rpm in an engine in air under standard conditions, at what angular velocity should a 1/3-scale model be spun in air at the same conditions to achieve dynamic similarity?

3. What is the ratio $P_{\text{real}}/P_{\text{model}}$ in these tests?
Problem 2:

Consider the two-dimensional, laminar, fully developed flow of water of uniform depth $d$ down a plane inclined at an angle $\alpha$ to the horizontal. The velocity at the water surface is $U$.

1. Use the differential $x$-momentum equation to find the velocity profile.

2. Find the shear stress at the wall: i) with a control volume analysis; and ii) using the velocity profile.

3. Find the skin friction coefficient in terms of: i) $\alpha$ and the Froude number; and ii) the Reynolds number.
Problem 3:

Two circular coaxial liquid (density $\rho$) jets with speed $V$ interact as shown in the figure. Liquid leaves the interaction region, which is open to the atmosphere, in a conical sheet. Obtain an expression for the angle $\theta$ of the resulting flow terms of $d_2/d_1$. Also, compute the tension in the cable and indicate the $d_2/d_1$ range over which this calculation valid.
Problem 4:

Water (density = 1000 kg/m\(^3\)) flows from the pipe shown in the figure as a free jet and strikes a circular flat plate. The flow geometry shown is axisymmetric. Assuming, steady, incompressible and inviscid flow, determine the flow rate and the manometer reading \(H\).
Problem 5:

A submarine is 100 m below the sea surface, as shown below. If the pressure inside the submarine is atmospheric and its hinged hatch is square, find the force $F$ required to open it. Assume $\rho$ is 1000 kg/m$^3$ and $g = 9.81$ m/s/s.
Integral mass conservation equation:

\[ 0 = \frac{\partial}{\partial t} \int_{cv} \rho d\mathbf{V} + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} \]

Integral momentum equation:

\[ \mathbf{F} = \mathbf{F}_s + \mathbf{F}_b = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho d\mathbf{V} + \int_{cs} \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A} \]

Integral angular momentum equation:

\[ \sum (\mathbf{r} \times \mathbf{F}) = \frac{\partial}{\partial t} \int_{cv} (\mathbf{r} \times \mathbf{V}) \rho d\mathbf{V} + \int_{cs} (\mathbf{r} \times \mathbf{V}) \rho \mathbf{V} \cdot d\mathbf{A} \]

The Equation of Continuity

Rectangular Coordinates \((x, y, z)\):

\[ \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho v_x) + \frac{\partial}{\partial y} (\rho v_y) + \frac{\partial}{\partial z} (\rho v_z) = 0 \]

Cylindrical Coordinates \((r, \theta, z)\):

\[ \frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \rho v_r) + \frac{1}{r} \frac{\partial}{\partial \theta} (\rho v_{\theta}) + \frac{\partial}{\partial z} (\rho v_z) = 0 \]

Momentum Equations for Incompressible Newtonian Fluid with Constant Viscosity \((\mu)\)

Rectangular Coordinates \((x, y, z)\):

\[ \rho \left( \frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} \right) = \mu \left( \frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2} \right) - \frac{\partial p}{\partial x} + \rho g_x \]

\[ \rho \left( \frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z} \right) = \mu \left( \frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2} \right) - \frac{\partial p}{\partial y} + \rho g_y \]

\[ \rho \left( \frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} \right) = \mu \left( \frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2} \right) - \frac{\partial p}{\partial z} + \rho g_z \]

Cylindrical Coordinates \((r, \theta, z)\):

\[ \rho \left( \frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_y}{r} \frac{\partial v_r}{\partial \theta} + \frac{v_z}{r} \frac{\partial v_r}{\partial z} \right) = \mu \left( \frac{\partial}{\partial r} \left( \frac{\partial v_r}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v_r}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} - \frac{2}{r^2} \right) - \frac{\partial p}{\partial r} + \rho g_r \]

\[ \rho \left( \frac{\partial v_{\theta}}{\partial t} + v_r \frac{\partial v_{\theta}}{\partial r} + \frac{v_y}{r} \frac{\partial v_{\theta}}{\partial \theta} + \frac{v_z}{r} \frac{\partial v_{\theta}}{\partial z} \right) = \mu \left( \frac{\partial}{\partial \theta} \left( \frac{\partial v_{\theta}}{\partial \theta} \right) + \frac{1}{r^2} \frac{\partial^2 v_{\theta}}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial v_{\theta}}{\partial \theta} - \frac{2}{r^2} \right) - \frac{1}{r} \frac{\partial p}{\partial \theta} + \rho g_{\theta} \]

\[ \rho \left( \frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_y}{r} \frac{\partial v_z}{\partial \theta} + \frac{v_z}{r} \frac{\partial v_z}{\partial z} \right) = \mu \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v_z}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial v_z}{\partial \theta} \right] - \frac{\partial p}{\partial z} + \rho g_z \]
Table C-3  Components of the Stress Tensor for Newtonian Fluids

<table>
<thead>
<tr>
<th>Rectangular Coordinates $(x, y, z)$</th>
<th>Cylindrical Coordinates $(r, \theta, z)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_{xx} = \mu \left[ \frac{\partial v_x}{\partial x} - \frac{2}{3} (\nabla \cdot v) \right]$</td>
<td>$\tau_{rr} = \mu \left[ \frac{\partial v_r}{\partial r} - \frac{2}{3} (\nabla \cdot v) \right]$</td>
</tr>
<tr>
<td>$\tau_{yy} = \mu \left[ \frac{\partial v_y}{\partial y} - \frac{2}{3} (\nabla \cdot v) \right]$</td>
<td>$\tau_{\theta\theta} = \mu \left[ \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} - \frac{2}{3} (\nabla \cdot v) \right]$</td>
</tr>
<tr>
<td>$\tau_{zz} = \mu \left[ \frac{\partial v_z}{\partial z} - \frac{2}{3} (\nabla \cdot v) \right]$</td>
<td>$\tau_{zz} = \mu \left[ \frac{2}{3} (\nabla \cdot v) \right]$</td>
</tr>
<tr>
<td>$\tau_{xy} = \tau_{yx} = \mu \left[ \frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right]$</td>
<td>$\tau_{\theta r} = \tau_{r\theta} = \mu \left[ \frac{\partial v_\theta}{\partial r} r \frac{\partial}{\partial \theta} + \frac{1}{r} \frac{\partial v_r}{\partial \theta} \right]$</td>
</tr>
<tr>
<td>$\tau_{yz} = \tau_{zy} = \mu \left[ \frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y} \right]$</td>
<td>$\tau_{\theta z} = \tau_{z\theta} = \mu \left[ \frac{\partial v_\theta}{\partial z} + \frac{1}{r} \frac{\partial v_r}{\partial \theta} \right]$</td>
</tr>
<tr>
<td>$\tau_{zx} = \tau_{xz} = \mu \left[ \frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z} \right]$</td>
<td>$\tau_{rz} = \tau_{zr} = \mu \left[ \frac{\partial v_r}{\partial r} + \frac{\partial v_z}{\partial z} \right]$</td>
</tr>
</tbody>
</table>

$$(\nabla \cdot v) = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

$$(\nabla \cdot v) = \frac{1}{r} \frac{\partial}{\partial r} (rv_\theta) + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z}$$

**Fig. 1.4.** Drag coefficient for circular cylinders as a function of the Reynolds number

$C_D = \frac{\sqrt{D}}{D}$

**Fig. 1.5.** Drag coefficient for spheres as a function of the Reynolds number

Curve (1): Stokes’ theory, eqn. (6.10); curve (2): Oseen’s theory, eqn. (6.13)

$C_D = 24/R$